Solving Multiple Integrals Using Maple

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Abstract The multiple integral problem is closely related to probability theory and quantum field theory. This paper uses the mathematical software Maple for the auxiliary tool to study three types of multiple integrals. We can obtain the infinite series forms of these three types of multiple integrals by using differentiation with respect to a parameter, differentiation term by term, and integration term by term. In addition, we provide some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

Keywords Multiple Integrals, Infinite Series Forms, Differentiation with Respect to a Parameter, Differentiation Term by Term, Integration Term By Term, Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Mozart, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and

inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, [1-7] can be adopted as references.

The multiple integral problem is closely related with probability theory and quantum field theory, and can refer to [8-9]. For this reason, the evaluation and numerical calculation of multiple integrals is important. In this study, we evaluate the following three types of n -tuple integrals

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \cos \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \quad (1)$$

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \sin \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \quad (2)$$

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \exp \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \quad (3)$$

where n is a positive integer, m_i are non-negative integers, and β_i, λ_i are real numbers for all $i = 1, \dots, n$. We can obtain the infinite series forms of these three types of multiple integrals by using differentiation with respect to a parameter, differentiation term by term, and integration term by term; these are the major results of this study (i.e., Theorems 1-3). In addition, we obtain three corollaries from the three theorems. For the study of related multiple integral problems can refer to [10-25]. On the other hand, we propose some multiple integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods. The following is the flowchart of the research method used in this paper.

Figure 1. The flowchart of the research method used in this paper.

2. Main Results

Firstly, we introduce some notations and formulas used in this paper.

2.1. Notations

- i. $\prod_{i=1}^n c_i = c_1 \times c_2 \times \cdots \times c_n$, where n is a positive integer, c_i are real numbers for all $i = 1, \dots, n$.
- ii. $p! = p \times (p-1) \times \cdots \times 1$, where p is a positive integer, and $0! = 1$.

2.2. Formulas

$$(A) \cos u = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} u^{2k}, \text{ where } u \text{ is a real number.}$$

$$(B) \sin u = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} u^{2k+1}, \text{ where } u \text{ is a real number.}$$

$$(C) e^u = \sum_{k=0}^{\infty} \frac{1}{k!} u^k, \text{ where } u \text{ is a real number.}$$

Next, we introduce three important theorems used in this study.

2.3. Differentiation with Respect to a Parameter ([26]).

Suppose $I = [c_1, c_2] \times [a_{11}, a_{12}] \times [a_{21}, a_{22}] \times \cdots \times [a_{n1}, a_{n2}]$, and the $(n+1)$ -variables function $f(\rho, x_1, x_2, \dots, x_n)$ is defined on I . If $f(\rho, x_1, x_2, \dots, x_n)$ and its partial derivative $\frac{\partial f}{\partial \rho}(\rho, x_1, x_2, \dots, x_n)$ are continuous functions on I . Then

$$F(\rho) = \int_{a_{11}}^{a_{12}} \int_{a_{21}}^{a_{22}} \cdots \int_{a_{n1}}^{a_{n2}} f(\rho, x_1, x_2, x_3, \dots, x_n) dx_1 dx_2 \cdots dx_n$$

is differentiable on the open interval (c_1, c_2) , and its derivative

$$\frac{d}{d\rho} F(\rho) = \int_{a_{11}}^{a_{12}} \int_{a_{21}}^{a_{22}} \cdots \int_{a_{n1}}^{a_{n2}} \frac{\partial f}{\partial \rho}(\rho, x_1, x_2, x_3, \dots, x_n) dx_2 dx_3 \cdots dx_n$$

for all $\rho \in (c_1, c_2)$.

2.4. Differentiation Term by Term ([27, p230])

For all non-negative integers k , if the functions $g_k : (a, b) \rightarrow \mathbb{R}$ satisfy the following three conditions : (i) there exists a point $x_0 \in (a, b)$ such that $\sum_{k=0}^{\infty} g_k(x_0)$ is convergent, (ii) all functions $g_k(x)$ are differentiable on open interval (a, b) , (iii) $\sum_{k=0}^{\infty} \frac{d}{dx} g_k(x)$ is uniformly convergent on (a, b) . Then $\sum_{k=0}^{\infty} g_k(x)$ is uniformly convergent and differentiable on (a, b) , and its derivative

$$\frac{d}{dx} \sum_{k=0}^{\infty} g_k(x) = \sum_{k=0}^{\infty} \frac{d}{dx} g_k(x).$$

2.5. Integration Term by Term ([27, p269]).

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an interval I . If $\sum_{n=0}^{\infty} \int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty} \int_I g_n$.

The following is the first major result in this study, we determine the infinite series form of the multiple integral (1).

2.6. Theorem 1 Suppose n is a positive integer, m_i are non-negative integers, and β_i, λ_i are real numbers, $\lambda_i \geq 0$, $\beta_i > -1$ for all $i = 1, \dots, n$. Then the n -tuple integral

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \cos \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \\ &= (-1)^{i=1} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)! \cdot \prod_{i=1}^n (2\lambda_i k + \beta_i + 1)^{m_i+1}} \end{aligned} \quad (4)$$

Proof Because

$$\begin{aligned} & \prod_{i=1}^n x_i^{\beta_i} \cdot \cos \left(\prod_{i=1}^n x_i^{\lambda_i} \right) \\ &= \prod_{i=1}^n x_i^{\beta_i} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \left(\prod_{i=1}^n x_i^{\lambda_i} \right)^{2k} \end{aligned}$$

(By Formula (A))

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \prod_{i=1}^n x_i^{2\lambda_i k + \beta_i} \quad (5)$$

Therefore,

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{\beta_i} \cdot \cos \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \\ &= \int_0^1 \cdots \int_0^1 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \prod_{i=1}^n x_i^{2\lambda_i k + \beta_i} dx_1 \cdots dx_n \quad (\text{By (5)}) \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{2\lambda_i k + \beta_i} dx_1 \cdots dx_n \end{aligned}$$

(By integration term by term)

$$\begin{aligned} &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \cdot \prod_{i=1}^n \int_0^1 x_i^{2\lambda_i k + \beta_i} dx_i \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)! \cdot \prod_{i=1}^n (2\lambda_i k + \beta_i + 1)} \quad (6) \end{aligned}$$

Using differentiation with respect to a parameter and differentiation term by term, we differentiate β_i by m_i times on both sides of (6) for all $i = 1, \dots, n$. We obtain

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \cos \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \\ &= (-1)^{\sum_{i=1}^n m_i} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)! \cdot \prod_{i=1}^n (2\lambda_i k + \beta_i + 1)^{m_i+1}} \blacksquare \end{aligned}$$

In Theorem 1, taking $x_i = e^{-y_i}$ for all $i = 1, \dots, n$, we immediately have the following result.

2.7. Corollary 1 If the assumptions are the same as Theorem 1, then the n -tuple improper integral

$$\begin{aligned} & \int_0^{\infty} \cdots \int_0^{\infty} \prod_{i=1}^n y_i^{m_i} \cdot \exp \left(- \sum_{i=1}^n (\beta_i + 1) y_i \right) \cos \left(\exp \left(- \sum_{i=1}^n \lambda_i y_i \right) \right) dy_1 \cdots dy_n \\ &= \prod_{i=1}^n m_i! \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)! \cdot \prod_{i=1}^n (2\lambda_i k + \beta_i + 1)^{m_i+1}} \quad (7) \end{aligned}$$

Next, we determine the infinite series forms of the multiple integral (2).

2.8. Theorem 2 Suppose n is a positive integer, m_i are non-negative integers, and β_i, λ_i are real numbers, $\lambda_i \geq 0$, $\lambda_i + \beta_i > -1$ for all $i = 1, \dots, n$. Then the n -tuple integral

$$\int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \sin \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n$$

$$= (-1)^{\sum_{i=1}^n m_i} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)! \cdot \prod_{i=1}^n (2\lambda_i k + \lambda_i + \beta_i + 1)^{m_i+1}} \quad (8)$$

Proof Because

$$\begin{aligned} & \prod_{i=1}^n x_i^{\beta_i} \cdot \sin \left(\prod_{i=1}^n x_i^{\lambda_i} \right) \\ &= \prod_{i=1}^n x_i^{\beta_i} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left(\prod_{i=1}^n x_i^{\lambda_i} \right)^{2k+1} \end{aligned}$$

(By Formula (B))

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \prod_{i=1}^n x_i^{2\lambda_i k + \lambda_i + \beta_i} \quad (9)$$

Thus,

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{\beta_i} \cdot \sin \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \\ &= \int_0^1 \cdots \int_0^1 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \prod_{i=1}^n x_i^{2\lambda_i k + \lambda_i + \beta_i} dx_1 \cdots dx_n \end{aligned}$$

(By (9))

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{2\lambda_i k + \lambda_i + \beta_i} dx_1 \cdots dx_n$$

(Using integration term by term)

$$\begin{aligned} &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot \prod_{i=1}^n \int_0^1 x_i^{2\lambda_i k + \lambda_i + \beta_i} dx_i \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)! \cdot \prod_{i=1}^n (2\lambda_i k + \lambda_i + \beta_i + 1)} \quad (10) \end{aligned}$$

Also, by differentiation with respect to a parameter and differentiation term by term, differentiating β_i by m_i times on both sides of (10) for all $i = 1, \dots, n$. We obtain

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \sin \left(\prod_{i=1}^n x_i^{\lambda_i} \right) dx_1 \cdots dx_n \\ &= (-1)^{\sum_{i=1}^n m_i} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)! \cdot \prod_{i=1}^n (2\lambda_i k + \lambda_i + \beta_i + 1)^{m_i+1}} \blacksquare \end{aligned}$$

In Theorem 2, let $x_i = e^{-y_i}$ for $i = 1, \dots, n$, then the following result holds.

2.9. Corollary 2 If the assumptions are the same as Theorem 2, then the n -tuple improper integral

$$\begin{aligned} & \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^n y_i^{m_i} \cdot \exp\left(-\sum_{i=1}^n (\beta_i + 1)y_i\right) \sin\left(\exp\left(-\sum_{i=1}^n \lambda_i y_i\right)\right) dy_1 \cdots dy_n \\ &= \prod_{i=1}^n m_i! \cdot \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)! \prod_{i=1}^n (2\lambda_i k + \lambda_i + \beta_i + 1)^{m_i+1}} \end{aligned} \quad (11)$$

Finally, we obtain the infinite series forms of the multiple integral (3).

2.10. Theorem 3 If the assumptions are the same as Theorem 1. Then the n -tuple integral

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \exp\left(\prod_{i=1}^n x_i^{\lambda_i}\right) dx_1 \cdots dx_n \\ &= (-1)^{\sum_{i=1}^n m_i} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^\infty \frac{1}{k! \prod_{i=1}^n (\lambda_i k + \beta_i + 1)^{m_i+1}} \end{aligned} \quad (12)$$

Proof Because

$$\begin{aligned} & \prod_{i=1}^n x_i^{\beta_i} \cdot \exp\left(\prod_{i=1}^n x_i^{\lambda_i}\right) \\ &= \prod_{i=1}^n x_i^{\beta_i} \cdot \sum_{k=0}^\infty \frac{1}{k!} \left(\prod_{i=1}^n x_i^{\lambda_i}\right)^k \end{aligned}$$

(Using Formula (C))

$$= \sum_{k=0}^\infty \frac{1}{k!} \prod_{i=1}^n x_i^{\lambda_i k + \beta_i} \quad (13)$$

Hence,

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{\beta_i} \cdot \exp\left(\prod_{i=1}^n x_i^{\lambda_i}\right) dx_1 \cdots dx_n \\ &= \int_0^1 \cdots \int_0^1 \sum_{k=0}^\infty \frac{1}{k!} \prod_{i=1}^n x_i^{\lambda_i k + \beta_i} dx_1 \cdots dx_n \quad (\text{By (13)}) \\ &= \sum_{k=0}^\infty \frac{1}{k!} \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{\lambda_i k + \beta_i} dx_1 \cdots dx_n \end{aligned}$$

(By integration term by term)

$$\begin{aligned} &= \sum_{k=0}^\infty \frac{1}{k!} \cdot \prod_{i=1}^n \int_0^1 x_i^{\lambda_i k + \beta_i} dx_i \\ &= \sum_{k=0}^\infty \frac{1}{k! \prod_{i=1}^n (\lambda_i k + \beta_i + 1)} \end{aligned} \quad (14)$$

By differentiation with respect to a parameter and differentiation term by term, differentiating β_i by m_i times on both sides of (14) for all $i = 1, \dots, n$. We have

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 \prod_{i=1}^n (\ln x_i)^{m_i} \cdot \prod_{i=1}^n x_i^{\beta_i} \cdot \exp\left(\prod_{i=1}^n x_i^{\lambda_i}\right) dx_1 \cdots dx_n \\ &= (-1)^{\sum_{i=1}^n m_i} \prod_{i=1}^n m_i! \cdot \sum_{k=0}^\infty \frac{1}{k! \prod_{i=1}^n (\lambda_i k + \beta_i + 1)^{m_i+1}} \quad \blacksquare \end{aligned}$$

In Theorem 3, let $x_i = e^{-y_i}$ for $i = 1, \dots, n$, the following result holds.

2.11. Corollary 3 If the assumptions are the same as Theorem 1, then the n -tuple improper integral

$$\begin{aligned} & \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^n y_i^{m_i} \cdot \exp\left(-\sum_{i=1}^n (\beta_i + 1)y_i\right) \exp\left(\exp\left(-\sum_{i=1}^n \lambda_i y_i\right)\right) dy_1 \cdots dy_n \\ &= \prod_{i=1}^n m_i! \cdot \sum_{k=0}^\infty \frac{1}{k! \prod_{i=1}^n (\lambda_i k + \beta_i + 1)^{m_i+1}} \end{aligned} \quad (15)$$

3. Examples

In the following, for the three types of multiple integrals in this study, we provide some examples and use Theorems 1-3 and Corollaries 1-3 to determine the infinite series forms of these multiple integrals. On the other hand, we employ Maple to calculate the approximations of these multiple integrals and their solutions for verifying our answers.

3.1. Example 1 By Theorem 1, we obtain the double integral

$$\begin{aligned} & \int_0^1 \int_0^1 (\ln x_1)^2 (\ln x_2)^3 x_1^{7/3} x_2^{9/2} \cos(x_1^{2/5} x_2^{11/6}) dx_1 dx_2 \\ &= -12 \cdot \sum_{k=0}^\infty \frac{(-1)^k}{(2k)! (4k/5 + 10/3)^3 (11k/3 + 11/2)^4} \end{aligned} \quad (16)$$

Next, we use Maple to verify the correctness of (16).

```
>evalf(Doubleint((ln(x1))^2*(ln(x2))^3*x1^(7/3)*x2^(9/2)
*cos(x1^(2/5)*x2^(11/6)),x1=0..1,x2=0..1),14);
-0.00034219267849328
```

```
>evalf(-12*sum((-1)^k/((2*k)!*(4*k/5+10/3)^3*(11*k/3+
11/2)^4),k=0..infinity),14);
-0.00034219267849327
```

3.2. Example 2 By Corollary 1, the triple improper integral

$$\begin{aligned} & \int_0^\infty \int_0^\infty \int_0^\infty y_1^3 y_2^5 y_3^2 \exp(-2y_1 - 3y_2 - 4y_3) \cos(\exp(-4y_1 - y_2 - 2y_3)) dy_1 dy_2 dy_3 \\ &= 1440 \cdot \sum_{k=0}^\infty \frac{(-1)^k}{(2k)! (8k+2)^4 (2k+3)^6 (4k+4)^3} \end{aligned} \quad (17)$$

```
>evalf(Tripleint(y1^3*y2^5*y3^2*exp(-2*y1-3*y2-4*y3)*
cos(exp(-4*y1-y2-2*y3)),y1=0..infinity,y2=0..infinity,y3=0
```

..infinity),14);

0.0019290033484884

>evalf(1440*sum((-1)^k/((2*k)!*(8*k+2)^4*(2*k+3)^6*(4*k+4)^3),k=0..infinity),14);

0.0019290033484884

3.3. Example 3 Using Theorem 2, we can evaluate the double integral

$$\int_0^1 \int_0^1 (\ln x_1)^5 (\ln x_2)^3 x_1^{11/7} x_2^{9/5} \sin(x_1^{6/7} x_2^{4/5}) dx_1 dx_2$$

$$= 720 \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(12k/7+24/7)^6 (8k/5+18/5)^4} \quad (18)$$

>evalf(Doubleint((ln(x1))^5*(ln(x2))^3*x1^(11/7)*x2^(9/5)*sin(x1^(6/7)*x2^(4/5)),x1=0..1,x2=0..1),14);

0.0026301802197270

>evalf(720*sum((-1)^k/((2*k+1)!*(12*k/7+24/7)^6*(8*k/5+18/5)^4),k=0..infinity),14);

0.0026301802197270

3.4. Example 4 By Corollary 2, the double improper integral

$$\int_0^{\infty} \int_0^{\infty} y_1^4 y_2^2 \exp(-2y_1 - 7y_2) \sin(\exp(-5y_1 - 3y_2)) dy_1 dy_2$$

$$= 48 \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(10k+7)^5 (6k+10)^3} \quad (19)$$

>evalf(Doubleint(y1^4*y2^2*exp(-2*y1-7*y2)*sin(exp(-5*y1-3*y2)),y1=0..infinity,y2=0..infinity),14);

0.0000028545799098894

>evalf(48*sum((-1)^k/((2*k+1)!*(10*k+7)^5*(6*k+10)^3),k=0..infinity),14);

0.0000028545799098893

3.5. Example 5 Using Theorem 3, the double integral

$$\int_0^1 \int_0^1 (\ln x_1)^2 (\ln x_2)^7 x_1^{1/4} x_2^{3/8} \exp(x_1^{4/9} x_2^{5/11}) dx_1 dx_2$$

$$= -10080 \cdot \sum_{k=0}^{\infty} \frac{1}{k! \cdot (4k/9+5/4)^3 (5k/11+11/8)^8} \quad (20)$$

>evalf(Doubleint((ln(x1))^2*(ln(x2))^7*x1^(1/4)*x2^(3/8)*exp(x1^(4/9)*x2^(5/11)),x1=0..1,x2=0..1),14);

-421.16580400909

>evalf(-10080*sum(1/(k!*(4*k/9+5/4)^3*(5*k/11+11/8)^8),k=0..infinity),14);

-421.16580400908

3.6. Example 6 By Corollary 3, we obtain the double improper integral

$$\int_0^{\infty} \int_0^{\infty} y_1 y_2^4 \exp(-2y_1 - 3y_2) \exp(\exp(-4y_1 - 2y_2)) dy_1 dy_2$$

$$= 24 \cdot \sum_{k=0}^{\infty} \frac{1}{k! \cdot (4k+2)^2 (2k+3)^5} \quad (21)$$

>evalf(Doubleint(y1*y2^4*exp(-2*y1-3*y2)*exp(exp(-4*y1-2*y2)),y1=0..infinity,y2=0..infinity),14);

0.024912197200303

>evalf(24*sum(1/(k!*(4*k+2)^2*(2*k+3)^5),k=0..infinity),14);

0.024912197200303

4. Conclusions

As mentioned, evaluating the multiple integrals is important in probability theory and quantum field theory. In this study, we provide a new technique to solve three types of multiple integrals, and we hope this method can be applied in mathematical statistics or quantum physics. Simultaneously, the differentiation with respect to a parameter, the differentiation term by term and the integration term by term play significant roles in the theoretical inferences of this study. In fact, the application of the three theorems is extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications.

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