A Study of Multiple Integrals with Maple

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Abstract The multiple integral problem is closely related with probability theory and quantum field theory. This paper uses the mathematical software Maple for the auxiliary tool to study two types of multiple integrals. We can obtain the infinite series forms of these two types of multiple integrals by using integration term by term theorem. In addition, we provide some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

Keywords Multiple Integrals, Infinite Series Forms, Integration Term By Term Theorem, Maple

1. Introduction

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. As for the instructions and operations of Maple, we can refer to [1-7].

The multiple integral problem is closely related with

probability theory and quantum field theory, and can refer to [8-9]. For this reason, the evaluation and numerical calculation of multiple integrals is important. In this study, we evaluate the following two types of n -tuple integrals

$$\int_0^1 \cdots \int_0^1 f\left(\prod_{m=1}^n x_m^{r_m}\right) dx_1 \cdots dx_n \quad (1)$$

$$\int_1^\infty \cdots \int_1^\infty g\left(\prod_{m=1}^n x_m^{s_m}\right) dx_1 \cdots dx_n \quad (2)$$

Where n is any positive integer, r_m, s_m are real numbers, $r_m \geq 0, s_m > 1$ for $m = 1, \dots, n$. And

$$f(t) = \frac{A_1}{t+a_1} + \cdots + \frac{A_p}{t+a_p} \quad (3)$$

$$g(t) = \frac{B_1}{t+b_1} + \cdots + \frac{B_p}{t+b_p} \quad (4)$$

Where P is a positive integer, A_j, B_j, a_j, b_j are real numbers, and $|a_j| > 1, |b_j| < 1$ for all $j = 1, \dots, p$. We can obtain the infinite series forms of these two types of multiple integrals by using integration term by term theorem; these are the major results of this study (i.e., Theorems 1 and 2). In addition, we obtain two corollaries from these two theorems. For the study of related multiple integral problems can refer to [10-23]. On the other hand, we propose some multiple integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce a notation and a formula used in this paper.

2.1. Notation

$\prod_{m=1}^n c_m = c_1 \times c_2 \times \cdots \times c_n$, where n is a positive integer, c_m are real numbers for $m = 1, \dots, n$.

2.2. Geometric Series

$\frac{1}{1+\beta} = \sum_{k=0}^{\infty} (-1)^k \beta^k$, where β is a real number, $|\beta| < 1$.

Next, we introduce an important theorem used in this study.

2.3. Integration Term by Term Theorem ([24]).

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an interval I . If $\sum_{n=0}^{\infty} \int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty} \int_I g_n$.

The following is the first major result in this study, we determine the infinite series form of the multiple integral (1).

2.4. Theorem 1

If n is a positive integer, r_m are real numbers, $r_m \geq 0$ for all $m = 1, \dots, n$. Suppose p is a positive integer, A_j, a_j are real numbers, and $|a_j| > 1$ for all $j = 1, \dots, p$. Let the domain of the function

$$f(t) = \frac{A_1}{t+a_1} + \cdots + \frac{A_p}{t+a_p}$$

be $\{t \in \mathbb{R} \mid |t| < 1\}$. Then the n -tuple integral

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 f\left(\prod_{m=1}^n x_m^{r_m}\right) dx_1 \cdots dx_n \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{\prod_{m=1}^n (r_m k + 1)} \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \end{aligned} \quad (5)$$

2.4.1. Proof

Because

$$\begin{aligned} f(t) &= \frac{A_1}{t+a_1} + \cdots + \frac{A_p}{t+a_p} \\ &= \frac{A_1}{a_1} \cdot \frac{1}{1+\frac{t}{a_1}} + \cdots + \frac{A_p}{a_p} \cdot \frac{1}{1+\frac{t}{a_p}} \end{aligned}$$

$$= \frac{A_1}{a_1} \cdot \sum_{k=0}^{\infty} (-1)^k \left(\frac{t}{a_1}\right)^k + \cdots + \frac{A_p}{a_p} \cdot \sum_{k=0}^{\infty} (-1)^k \left(\frac{t}{a_p}\right)^k$$

(By geometric series because $|t| < 1$ and $|a_j| > 1$ for all $j = 1, \dots, p$)

$$= \sum_{k=0}^{\infty} (-1)^k \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) t^k \quad (6)$$

Therefore,

$$\begin{aligned} & \int_0^1 \cdots \int_0^1 f\left(\prod_{m=1}^n x_m^{r_m}\right) dx_1 \cdots dx_n \\ &= \lim_{\substack{\delta_m \rightarrow 1^- \\ 1 \leq m \leq n}} \int_0^{\delta_n} \cdots \int_0^{\delta_1} f\left(\prod_{m=1}^n x_m^{r_m}\right) dx_1 \cdots dx_n \\ &= \lim_{\substack{\delta_m \rightarrow 1^- \\ 1 \leq m \leq n}} \int_0^{\delta_n} \cdots \int_0^{\delta_1} \sum_{k=0}^{\infty} (-1)^k \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \left(\prod_{m=1}^n x_m^{r_m} \right)^k dx_1 \cdots dx_n \\ &\quad \text{(Using (6))} \\ &= \lim_{\substack{\delta_m \rightarrow 1^- \\ 1 \leq m \leq n}} \sum_{k=0}^{\infty} (-1)^k \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \int_0^{\delta_n} \cdots \int_0^{\delta_1} \prod_{m=1}^n x_m^{r_m k} dx_1 \cdots dx_n \\ &\quad \text{(By integration term by term theorem)} \\ &= \lim_{\substack{\delta_m \rightarrow 1^- \\ 1 \leq m \leq n}} \sum_{k=0}^{\infty} (-1)^k \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \prod_{m=1}^n \int_0^{\delta_m} x_m^{r_m k} dx_m \\ &= \lim_{\substack{\delta_m \rightarrow 1^- \\ 1 \leq m \leq n}} \sum_{k=0}^{\infty} (-1)^k \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \prod_{m=1}^n \frac{\delta_m^{r_m k + 1}}{r_m k + 1} \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{\prod_{m=1}^n (r_m k + 1)} \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \blacksquare \end{aligned}$$

In Theorem 1, taking $x_m = e^{-w_m}$ for all $m = 1, \dots, n$, we immediately have the following result.

2.5. Corollary 1

If the assumptions are the same as Theorem 1, then the n -tuple improper integral

$$\begin{aligned} & \int_0^{\infty} \cdots \int_0^{\infty} \exp\left(-\sum_{m=1}^n w_m\right) f\left(\exp\left(-\sum_{m=1}^n r_m w_m\right)\right) dw_1 \cdots dw_n \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{\prod_{m=1}^n (r_m k + 1)} \left(\sum_{j=1}^p \frac{A_j}{a_j^{k+1}} \right) \end{aligned} \quad (7)$$

The following is the second major result in this paper, we obtain the infinite series forms of the multiple integral (2).

2.6. Theorem 2

If n is a positive integer, s_m are real numbers, $s_m > 1$ for all $m = 1, \dots, n$. Suppose P is a positive integer, B_j, b_j are real numbers, and $|b_j| < 1$ for all $j = 1, \dots, P$. Suppose the domain of the function

$$g(t) = \frac{B_1}{t + b_1} + \dots + \frac{B_P}{t + b_P}$$

is $\{t \in \mathbb{R} \mid |t| > 1\}$. Then the n -tuple improper integral

$$\begin{aligned} & \int_1^\infty \dots \int_1^\infty g\left(\prod_{m=1}^n x_m^{s_m}\right) dx_1 \dots dx_n \\ &= \sum_{k=0}^\infty \frac{(-1)^k}{\prod_{m=1}^n (s_m k + s_m - 1)} \left(\sum_{j=1}^P B_j b_j^k\right) \end{aligned} \quad (8)$$

2.6.1. Proof

Because

$$\begin{aligned} g(t) &= \frac{B_1}{t + b_1} + \dots + \frac{B_P}{t + b_P} \\ &= \frac{B_1}{t} \cdot \frac{1}{1 + \frac{b_1}{t}} + \dots + \frac{B_P}{t} \cdot \frac{1}{1 + \frac{b_P}{t}} \\ &= \frac{B_1}{t} \cdot \sum_{k=0}^\infty (-1)^k \left(\frac{b_1}{t}\right)^k + \dots + \frac{B_P}{t} \cdot \sum_{k=0}^\infty (-1)^k \left(\frac{b_P}{t}\right)^k \end{aligned}$$

(Using geometric series because $|t| > 1$ and $|b_j| < 1$ for all $j = 1, \dots, P$)

$$= \sum_{k=0}^\infty (-1)^k \left(\sum_{j=1}^P B_j b_j^k\right) t^{-k-1} \quad (9)$$

Thus,

$$\begin{aligned} & \int_1^\infty \dots \int_1^\infty g\left(\prod_{m=1}^n x_m^{s_m}\right) dx_1 \dots dx_n \\ &= \lim_{\substack{\lambda_m \rightarrow 1^+ \\ 1 \leq m \leq n}} \int_{\lambda_n}^\infty \dots \int_{\lambda_1}^\infty g\left(\prod_{m=1}^n x_m^{s_m}\right) dx_1 \dots dx_n \\ &= \lim_{\substack{\lambda_m \rightarrow 1^+ \\ 1 \leq m \leq n}} \int_{\lambda_n}^\infty \dots \int_{\lambda_1}^\infty \sum_{k=0}^\infty (-1)^k \left(\sum_{j=1}^P B_j b_j^k\right) \left(\prod_{m=1}^n x_m^{s_m}\right)^{-k-1} dx_1 \dots dx_n \end{aligned}$$

(By (9))

$$= \lim_{\substack{\lambda_m \rightarrow 1^+ \\ 1 \leq m \leq n}} \sum_{k=0}^\infty (-1)^k \left(\sum_{j=1}^P B_j b_j^k\right) \int_{\lambda_n}^\infty \dots \int_{\lambda_1}^\infty \prod_{m=1}^n x_m^{-s_m k - s_m} dx_1 \dots dx_n$$

(Using integration term by term theorem)

$$\begin{aligned} &= \lim_{\substack{\lambda_m \rightarrow 1^+ \\ 1 \leq m \leq n}} \sum_{k=0}^\infty (-1)^k \left(\sum_{j=1}^P B_j b_j^k\right) \prod_{m=1}^n \int_{\lambda_m}^\infty x_m^{-s_m k - s_m} dx_m \\ &= \lim_{\substack{\lambda_m \rightarrow 1^+ \\ 1 \leq m \leq n}} \sum_{k=0}^\infty (-1)^k \left(\sum_{j=1}^P B_j b_j^k\right) \prod_{m=1}^n \frac{\lambda_m^{-s_m k - s_m + 1}}{-s_m k - s_m + 1} \\ &= \sum_{k=0}^\infty \frac{(-1)^k}{\prod_{m=1}^n (s_m k + s_m - 1)} \left(\sum_{j=1}^P B_j b_j^k\right) \end{aligned}$$

In Theorem 2, let $x_m = e^{u_m}$ for all $m = 1, \dots, n$, then the following result holds.

2.7. Corollary 2

If the assumptions are the same as Theorem 2, then the n -tuple improper integral

$$\begin{aligned} & \int_0^\infty \dots \int_0^\infty \exp\left(\sum_{m=1}^n u_m\right) g\left(\exp\left(\sum_{m=1}^n s_m u_m\right)\right) du_1 \dots du_n \\ &= \sum_{k=0}^\infty \frac{(-1)^k}{\prod_{m=1}^n (s_m k + s_m - 1)} \left(\sum_{j=1}^P B_j b_j^k\right) \end{aligned} \quad (10)$$

3. Examples

In the following, for the two types of multiple integrals in this study, we provide some examples and use Theorems 1, 2 and Corollaries 1, 2 to determine the infinite series forms of these multiple integrals. On the other hand, we employ Maple to calculate the approximations of these multiple integrals and their solutions for verifying our answers.

3.1. Example 1

Let the domain of the function

$$f_1(t) = \frac{t^2 + 33t - 118}{t^3 - 5t^2 - 2t + 24} \quad (11)$$

be $\{t \in \mathbb{R} \mid |t| < 1\}$, and let $r_1, r_2, r_3 \geq 0$. Because

$$f_1(t) = \frac{5}{t-4} + \frac{-6}{t+2} + \frac{2}{t-3} \quad (12)$$

(The case of $A_1 = 5, A_2 = -6, A_3 = 2, a_1 = -4, a_2 = 2, a_3 = -3$ in Theorem 1). Thus, by Theorem 1, we obtain the triple integral

$$\int_0^1 \int_0^1 \int_0^1 \frac{x_1^2 x_2^2 x_3^2 + 33x_1 x_2^2 x_3^2 - 118}{x_1^3 x_2^3 x_3^3 - 5x_1^2 x_2^2 x_3^3 - 2x_1 x_2^2 x_3^3 + 24} dx_1 dx_2 dx_3$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(r_1 k + 1)(r_2 k + 1)(r_3 k + 1)} \left(\frac{5}{(-4)^{k+1}} - \frac{6}{2^{k+1}} + \frac{2}{(-3)^{k+1}} \right) \quad (13)$$

Hence, we obtain

$$\int_0^1 \int_0^1 \int_0^1 \frac{x_1^2 x_2^4 x_3^6 + 33x_1 x_2^2 x_3^3 - 118}{x_1^3 x_2^6 x_3^9 - 5x_1^2 x_2^4 x_3^6 - 2x_1 x_2^2 x_3^3 + 24} dx_1 dx_2 dx_3$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)(2k+1)(3k+1)} \left(\frac{5}{(-4)^{k+1}} - \frac{6}{2^{k+1}} + \frac{2}{(-3)^{k+1}} \right) \quad (14)$$

In the following, we use Maple to verify the correctness of (14).

```
>evalf(Tripleint((x1^2*x2^4*x3^6+33*x1*x2^2*x3^3-118)
/(x1^3*x2^6*x3^9-5*x1^2*x2^4*x3^6-2*x1*x2^2*x3^3+2
4),x1=0..1,x2=0..1,x3=0..1),14);
-4.8841370709055
>evalf(sum((-1)^k/((k+1)*(2*k+1)*(3*k+1))*(5/(-4)^(k+1)-
6/2^(k+1)+2/(-3)^(k+1)),k=0..infinity),14);
-4.8841370709060
```

3.2. Example 2

Suppose the domain of the function

$$f_2(t) = \frac{-2t-22}{t^2-2t-8} \quad (15)$$

is $\{t \in R \mid |t| < 1\}$, and let $r_1, r_2 \geq 0$. Because

$$f_2(t) = \frac{3}{t+2} + \frac{-5}{t-4} \quad (16)$$

(The case of $A_1 = 3, A_2 = -5, a_1 = 2, a_2 = -4$ in Corollary 1). Therefore, by Corollary 1, we have the double improper integral

$$\int_0^{\infty} \int_0^{\infty} \frac{\exp(-w_1 - w_2)[-2\exp(-r_1 w_1 - r_2 w_2) - 22]}{\exp(-2r_1 w_1 - 2r_2 w_2) - 2\exp(-r_1 w_1 - r_2 w_2) - 8} dw_1 dw_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(r_1 k + 1)(r_2 k + 1)} \left(\frac{3}{2^{k+1}} - \frac{5}{(-4)^{k+1}} \right) \quad (17)$$

Hence, we can determine

$$\int_0^{\infty} \int_0^{\infty} \frac{\exp(-w_1 - w_2)[-2\exp(-2w_1 - w_2) - 22]}{\exp(-4w_1 - 2w_2) - 2\exp(-2w_1 - w_2) - 8} dw_1 dw_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(k+1)} \left(\frac{3}{2^{k+1}} - \frac{5}{(-4)^{k+1}} \right) \quad (18)$$

We also use Maple to verify the correctness of (18).

```
>evalf(Doubleint(exp(-w1-w2)*(-2*exp(-2*w1-w2)-22)/
(exp(-4*w1-2*w2)-2*exp(-2*w1-w2)-8),w1=0..infinity,w2
=0..infinity),14);
2.7029842891882
>evalf(sum((-1)^k/((2*k+1)*(k+1))*(3/2^(k+1)-5/(-4)^(k+1)
)),k=0..infinity),14);
2.7029842891882
```

3.3. Example 3

Assume the domain of the function

$$g_1(t) = \frac{29t-12}{6t^2-5t+1} \quad (19)$$

is $\{t \in R \mid |t| > 1\}$, and let $s_1, s_2 > 1$. Because

$$g_1(t) = \frac{5/2}{t-1/2} + \frac{7/3}{t-1/3} \quad (20)$$

(The case of $B_1 = 5/2, B_2 = 7/3, b_1 = -1/2, b_2 = -1/3$ in Theorem 2). Thus, using Theorem 2, we obtain the double improper integral

$$\int_1^{\infty} \int_1^{\infty} \frac{29x_1^{s_1} x_2^{s_2} - 12}{6x_1^{2s_1} x_2^{2s_2} - 5x_1^{s_1} x_2^{s_2} + 1} dx_1 dx_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(s_1 k + s_1 - 1)(s_2 k + s_2 - 1)} \left(\frac{5}{2} \left(-\frac{1}{2} \right)^k + \frac{7}{3} \left(-\frac{1}{3} \right)^k \right) \quad (21)$$

Therefore, we have

$$\int_1^{\infty} \int_1^{\infty} \frac{29x_1^4 x_2^3 - 12}{6x_1^8 x_2^6 - 5x_1^4 x_2^3 + 1} dx_1 dx_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k+3)(3k+2)} \left(\frac{5}{2} \left(-\frac{1}{2} \right)^k + \frac{7}{3} \left(-\frac{1}{3} \right)^k \right) \quad (22)$$

Using Maple to verify the correctness of (22) as follows:

```
>evalf(Doubleint((29*x1^4*x2^3-12)/(6*x1^8*x2^6-5*x1^4
4*x2^3+1),x1=1..infinity,x2=1..infinity),14);
0.87700403798300
>evalf(sum((-1)^k/((4*k+3)*(3*k+2))*(5/2*(-1/2)^k+7/3*(-
1/3)^k),k=0..infinity),16);
0.877004037982999
```

3.4. Example 4

Let the domain of the function

$$g_2(t) = \frac{19t+1}{12t^2+11t+2} \quad (23)$$

be $\{t \in R \mid |t| > 1\}$, and let $s_1, s_2 > 1$. Because

$$g_2(t) = \frac{7/3}{t+2/3} + \frac{-3/4}{t+1/4} \quad (24)$$

(The case of $B_1 = 7/3, B_2 = -3/4, b_1 = 2/3, b_2 = 1/4$ in Corollary 2). Hence, by Corollary 2, we obtain the double

improper integral

$$\int_0^\infty \int_0^\infty \frac{\exp(u_1 + u_2) \cdot [19 \exp(s_1 u_1 + s_2 u_2) + 1]}{12 \exp(2s_1 u_1 + 2s_2 u_2) + 11 \exp(s_1 u_1 + s_2 u_2) + 2} du_1 du_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(s_1 k + s_1 - 1)(s_2 k + s_2 - 1)} \left(\frac{7}{3} \cdot \left(\frac{2}{3} \right)^k - \frac{3}{4} \cdot \left(\frac{1}{4} \right)^k \right) \quad (25)$$

Thus, we can evaluate

$$\int_0^\infty \int_0^\infty \frac{\exp(u_1 + u_2) \cdot [19 \exp(2u_1 + 5u_2) + 1]}{12 \exp(4u_1 + 10u_2) + 11 \exp(2u_1 + 5u_2) + 2} du_1 du_2$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(5k+4)} \left(\frac{7}{3} \cdot \left(\frac{2}{3} \right)^k - \frac{3}{4} \cdot \left(\frac{1}{4} \right)^k \right) \quad (26)$$

Verifying the correctness of (26) as follows:

```
>evalf(Doubleint(exp(u1+u2)*(19*exp(2*u1+5*u2)+1)/(12
*exp(4*u1+10*u2)+11*exp(2*u1+5*u2)+2),u1=0..infinity,
u2=0..infinity),14);
0.35566818033965
>evalf(sum((-1)^k/((2*k+1)*(5*k+4))*(7/3*(2/3)^k-3/4*(1/
4)^k),k=0..infinity),14);
0.35566818033959
```

4. Conclusions

As mentioned, the multiple integral problem is important in probability theory and quantum field theory. In this study, we propose a new technique to solve two types of multiple integrals, and we hope this method can be applied in mathematical statistics or quantum physics. Simultaneously, we know the integration term by term theorem plays a significant role in the theoretical inferences of this paper. In fact, the application of this theorem is extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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