A Note on Domatic Subdivision Stable Graphs

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Abstract A domatic partition of a graph $G = (V, E)$ is a partition of V into disjoint sets $V_1, V_2, \dots, V_K$ such that each V_i is a dominating set for G . A subdivision of a graph G is a graph resulting from the subdivision of edges in G . In this paper we discuss about the minimal properties of domatic subdivision stable graph and we show that every graph is an induced subgraph of a domatic subdivision stable graph. We discuss methods of generating new domatic subdivision stable graphs from existing domatic subdivision stable graphs using graph operations.

Keywords Domatic Partition, Subdivision, Dominating Set, Minimal Set

1. Introduction

We consider only simple connected undirected graphs $G = (V, E)$. P_n denotes the path of length n . The open neighborhood of vertex $v \in V(G)$ is defined by $N(v) = \{u \in V(G) \mid uv \in E(G)\}$ while its closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. The private neighborhood of $v \in D$ is defined by $pn[v, D] = N(v) - N(D - \{v\})$. We say that H is a subgraph of G , if $V(H) \subseteq V(G)$ and $uv \in E(H)$ implies $uv \in E(G)$. If a subgraph H satisfies the added property that for every pair u, v of vertices, $uv \in E(H)$ if and only if $uv \in E(G)$, then H is called an induced subgraph of G and is denoted by $\langle H \rangle$. We indicate that u is adjacent to v by writing $u \perp v$. For properties related to graph theory we refer to West, D. B., [1].

A set of vertices D in a graph $G = (V, E)$ is a dominating set if every vertex of $V - D$ is adjacent to some vertex of D . If D has the smallest possible cardinality of any dominating set of G , then D is called a minimum dominating set — abbreviated MDS. The cardinality of any MDS for G is called the domination number of G and it is denoted by $\gamma(G)$. A γ -set denotes a dominating set for G with minimum cardinality. A dominating set D is minimal dominating if no proper subset of D is a dominating set. A vertex in $V - D$ is k -dominated if it is dominated by at least k vertices in D . For properties related to domination we refer to Haynes, T. W. et al., [3, 4].

In [2], Vestergaard, P. D. et al., have studied the domatic

numbers and total domatic numbers of graphs having cut vertices. In Theorem 1, 3 and 4 of [2], they have provided upper bounds related to domination numbers and provided conditions for attaining the sharp bounds.

In [6], Yamuna, M. et al., have defined domatic subdivision stable graphs. In Theorem 1, 2 and 3 of [6], they have provided bounds for domatic subdivision stable graphs and obtained necessary and sufficient conditions for the domatic partition of the domatic subdivision stable graphs.

2. Materials and Methods

A domatic partition of a graph $G = (V, E)$ is a partition of V into disjoint sets $V_1, V_2, \dots, V_K$ such that each V_i is a dominating set for G . The domatic number is the maximum number of such disjoint sets and it is denoted by $d(G)$.

A subdivision of a graph G is a graph resulting from the subdivision of edges in G . The subdivision of some edge e with endpoints $\{u, v\}$ yields a graph containing one new vertex w , and with an edge set replacing e by two new edges, uw and wv . We shall denote the graph obtained by subdividing any edge uv of a graph G , by $G_{sd} uv$. Let w be a vertex introduced by subdividing uv . We shall denote this by $G_{sd} uv = w$.

Yamuna, M and Karthika, K. [6], defined a new graph called domatic subdivision stable graph. A graph G is said to be domatic subdivision stable(dss), if $d(G) = d(G_{sd} uv)$, for all $uv \in E(G)$.

Example of Dss Graph

Figure 1. G is dss. Here $d(G) = |\{5, 6\}, \{8, 2, 4\}, \{1, 7, 3\}| = 3$ and $d(G_{sd} 58) = |\{4, w, 6\}, \{5, 2, 3\}, \{1, 7, 8\}| = 3$. This is true for all $uv \in E(G)$. Let $S = \{5, 6, 7, 8\} \subseteq V(G)$. $\langle S \rangle = C_4$ in G .

Yamuna, M and Karthika, K [6], have proved the following

results,

R₁. Let G be any graph. Then $d(G_{sd} uv) \leq 3$.

R₂. When $d(G) = 3$, $d(G_{sd} uv) = 3$ if and only if there is a partition $Z = \{V_1, V_2, V_3\}$ for G such that

- V_1 and V_2 are dominating sets for G such that $u \in V_1$, $v \in V_2$.
- v is 2-dominated with respect to V_1 .
- u is 2-dominated with respect to V_2 .
- V_3 dominates at least $G - \{u\} - \{v\}$.

R₃. When $d(G) = 2$, $d(G_{sd} uv) = 3$ if and only if there is a domatic partition $d(G) = \{V_1\}, \{V_2\}$, $u \in V_1$, $v \in V_2$ such that

- v is 2-dominated with respect to V_1 .
- $V_2 = V_{12} \cup V_{22}$, where
 - $v \in V_{12}$, u is 2-dominated with respect to V_{12} .
 - V_{22} does not dominate at least one of u or v , that is V_{22} is a dominating set for $G - \{u\} - \{v\}$.

3. Results and Discussion

Minimal Dominating Sets and dss Graphs

In this section we provide results relating domatic subdivision stable graphs and minimal properties of the dominating sets.

Theorem 1

Let $d(G) = 2$ and $d(G_{sd} uv) = 3$. There is a domatic partition for G such that at least one V_i , $i = 1, 2$ is not minimal.

Proof

Let $d(G) = 2$ and $d(G_{sd} uv) = 3 = |\{V_1\}, \{V_2\}, \{V_3\}|$. By result R₃, $V_1, V_2 \cup V_3 - \{w\}$ are dominating set for G , such that $V_2 \cup V_3 - \{w\}$ is not minimal, since V_2 itself is a dominating set.

If $|V_3 - \{w\}| \geq 2$, then $V_1 \cup V_{13}, V_2 \cup V_{23}$ are dominating set for G , where $\{V_{13}, V_{23}\}$ is a partition for $V_3 - \{w\}$. This implies, there is a partition for G say $d(G) = \{V_1 \cup V_{13}\}, \{V_2 \cup V_{23}\}$, such that both the sets in the partition are not minimal.

Theorem 2

If $d(G) = |\{V_1\}, \{V_2\}|$ such that V_1 and V_2 are both minimal for all possible domatic partition for G , then $d(G_{sd} uv) \neq 3$.

Proof

If $d(G_{sd} uv) = 3$, say $d(G_{sd} uv) = |\{U_1\}, \{U_2\}, \{U_3\}|$, then by R₁, let us assume that, $u \in U_1$, $v \in U_2$, $w \in U_3$. $U_1, U_2, U_3 - \{w\}$ are dominating sets for G also. Then $d(G) = |\{U_1\}, \{U_2\} \cup \{U_3 - \{w\}\}|$ is a domatic partition for G such that $U_2 \cup \{U_3 - \{w\}\}$ is not minimal, a contradiction to our assumption.

Theorem 3

When $d(G) = 2$, $d(G_{sd} uv) = 2$ if for all possible partition $d(G) = |\{V_1\}, \{V_2\}|$ if V_i is not minimal, $i = 1, 2$, then

- If V_{11} is a dominating set for G , V_{12} does not dominate $G - \{u\} - \{v\}$, where $V_i = V_{i1} \cup V_{i2}$.
- $V_{12} \cup V_{22}$ does not dominate $G - \{u\} - \{v\}$.

Proof

Let $d(G) = |\{V_1\}, \{V_2\}|$ such that at least one V_i is not minimal satisfying the conditions of the theorem.

Let $V_1 = \{V_{11} \cup V_{12}\}$ and $V_2 = \{V_{21} \cup V_{22}\}$. Assume that V_{11} and V_{21} are minimal dominating sets for G . Given that V_{12}, V_{22} and $V_{12} \cup V_{22}$ does not dominate $G - \{u\} - \{v\}$.

In this case, $d(G_{sd} uv) \neq 3$. Suppose $d(G_{sd} uv) = 3$, say $d(G_{sd} uv) = |\{U_1\}, \{U_2\}, \{U_3\}|$. Then as in Theorem 2, $d(G) = |\{U_1\}, \{U_2 \cup \{U_3 - \{w\}\}\}|$ is a domatic partition for G such that $U_3 - \{w\}$ dominates $G - \{u\} - \{v\}$, a contradiction to the hypothesis of the theorem. Since $d(G_{sd} uv) \neq 3$, $d(G_{sd} uv) = 2$.

Theorem 4

Let $d(G) = 2$ such that

- At least one of V_i is not minimal.
- V_{11}, V_{21} are minimal dominating sets for G , where $V_1 = \{V_{11} \cup V_{12}\}$ and $V_2 = \{V_{21} \cup V_{22}\}$.
- V_{12}, V_{22} dominates $G - \{u\} - \{v\}$.

Then there is at least one partition for G , where u, v does not belong to the same partition.

Proof

Assume that u, v belong to the same partition for every possible domatic partition for G , say $u, v \in V_1$, where $d(G) = |\{V_1\}, \{V_2\}|$.

Case i V_2 is not minimal

Let $V_2 = V_{21} \cup V_{22}$, where V_{21} is a dominating set and V_{22} dominates $G - \{u\} - \{v\}$. Let $V_3 = \{V_{22} \cup \{u\}\}$ and $V_4 = \{V_{21} \cup \{V_1 - \{u\}\}\}$.

Case ii V_1 is not minimal

Let $V_1 = V_{11} \cup V_{12}$, where V_{11} is a dominating set for G , V_{12} dominates $G - \{u\} - \{v\}$. Then $u, v \in V_{11}$. Let $V_3 = \{V_{12} \cup \{u\}\}$, $V_4 = \{V_{11} - \{u\} \cup \{V_2\}\}$.

Case iii V_1 and V_2 are both not minimal

Let $V_1 = \{V_{11} \cup V_{12}\}$ and $V_2 = \{V_{21} \cup V_{22}\}$, where V_{11} and V_{21} are minimal dominating sets for G . Let V_{12}, V_{22} dominate $G - \{u\} - \{v\}$, $u, v \in V_{11}$. Let $V_3 = \{V_{12} \cup \{u\}\}$, $V_4 = \{V_{11} - \{u\} \cup \{V_2\}\}$.

In all three cases $d(G) = |\{V_3\}, \{V_4\}|$ is a domatic partition for G such that $u \in V_3$, $v \in V_4$, a contradiction.

Hence there is at least one partition for G , where u, v does not belong to the same partition.

Theorem 5

Let $d(G) = 3$. If for any $u, v \in E(G)$, there is a domatic partition $d(G) = |\{V_1\}, \{V_2\}, \{V_3\}|$ such that

- $u \in V_1, v \in V_2$,
- u is selfish with respect to V_1 ,
- v is selfish with respect to V_2 ,

then $d(G_{sd} uv) = 3$.

Proof

Let $d(G) = 3$ and $u, v \in E(G)$. Let $d(G) = |\{V_1\}, \{V_2\}, \{V_3\}|$ such that the conditions of the theorem is satisfied. Then,

When $u \in V_1$, v is 2-dominated.

When $v \in V_2$, u is 2-dominated.

V_3 dominate $G - \{u\} - \{v\}$.

By R₂, $d(G_{sd} uv) = 3$. Also $d(G_{sd} uv) = |\{V_1\}, \{V_2\}, \{V_3 \cup \{w\}\}|$.

Let $u \in V(G_1)$, $v \in V(G_2)$ such that $d(G_1) = d(G_2) = k$. Let $u \in V(G_1)$, $v \in V(G_2)$ such that $d(G_1 - \{u\}) = d(G_1)$, $d(G_2 - \{v\}) = d(G_2)$. Let G be the graph obtained by merging vertex u and v .

Claim 1

If $d(G) = d(G - \{u\})$, for some $u \in V(G)$, then u cannot be a saturated vertex.

Proof

If possible assume that u is a saturated vertex. Since $d(G) = d(G - \{u\}) = k$ (say), there are k - dominating sets $V_1, V_2, \dots, V_k$ for $G - \{u\}$. Since u is a saturated vertex, $V_1, V_2, \dots, V_k$ are dominating sets for G also, which implies $V_1, V_2, \dots, V_k, u$ are all dominating sets for G . This implies $|d(G)| > k$, a contradiction. So, if $d(G) = d(G - \{u\})$, for any graph G , u is not a saturated vertex.

From Claim 1, we conclude that u and v are not saturated vertices with respect to G_1 and G_2 .

Since uv is a cut vertex, we observe that, if U_1 is a dominating set for G , then U_1 can be partition as

$$U_1 = \begin{cases} U_{11} \cup U_{12} \cup \{uv\}, & \text{if } uv \in U_1 \\ U_{11} \cup U_{12}, & \text{if } uv \notin U_1 \end{cases}$$

where $V(U_{11}) \subseteq V(G_1)$, $U_{12} \subseteq V(G_2)$.

Also we observe that, $U_{11} \cup \{u\}$, $U_{12} \cup \{v\}$ are dominating sets for G_1 and G_2 respectively, if $uv \in U_1$. U_{11} , U_{12} are dominating sets for G_1 and G_2 respectively, if $uv \notin U_1$. Since $d(G_1 - \{u\}) = d(G_2 - \{v\}) = k$, by R_4 , $d(G) \leq k + 1$.

Claim 2

$$d(G) \neq k + 1.$$

Proof

If possible, let $d(G) = k + 1 = |\{V_1\}, \{V_2\}, \dots, \{V_k\}, \{V_{k+1}\}|$. Let $V_i = V_{i1} \cup V_{i2}$, where $V_{i1} = V(G_1) - \{u\}$, $V_{i2} = V(G_2) - \{v\}$. Let $uv \in V_1$. By claim 1, there is atleast one vertex say $u_1 \in G_1$, $v_1 \in G_2$ such that $uv, u_1, v_1 \in V_1$. $\{V_{11} \cup \{u\}, V_{21}, \dots, V_{k1}, V_{(k+1)1}\}$ is a domatic partition for G_1 , since $V_{21}, V_{31}, \dots, V_{k1}, V_{(k+1)1}$ dominates G_1 , a contradiction as $d(G_1) = k$. This implies, $d(G) \neq k + 1$.

By claim 2, we conclude that $d(G) = k$.

Converse of Theorem 10 need not be true.

Example

Figure 3. $d(G) = d(G_1) = d(G_2) = 5$, but $d(G_1 - \{u\}) = d(G_2 - \{v\}) = 4$, that is $d(G_1) \neq d(G_1 - \{u\})$ and $d(G_2) \neq d(G_2 - \{v\})$.

Theorem 11

If G_1 and G_2 are dss graphs such that $d(G_1) = d(G_2) = 3$, $d(G_1) = d(G_1 - \{u\})$, $d(G_2) = d(G_2 - \{v\})$, for some $u \in V_1$, $v \in V_2$, then the graph G obtained by merging u and v is also dss.

Proof

By Theorem 10, we know that $d(G) = 3$. Let $G_{sd} xy = w$, for some $xy \in E(G)$. Let $e = xy \in E(G)$, either $e \in E(G_1)$ or $e \in E(G_2)$. Assume that $e \in E(G_1)$.

Since G_1 is dss, let $d(G_1 - \{u\}) = |\{U_1\}, \{U_2\}, \{U_3\}|$ such that $u \in U_1$. Since $d(G_2) = 3$, let $d(G_2) = |\{C_1\}, \{C_2\}, \{C_3\}|$ such that $v \in C_1$. Then $\{U_1 \cup C_1 - \{u\} - \{v\} \cup \{uv\}, U_2 \cup C_2, U_3 \cup C_3\}$ is a domatic partition for $G_{sd} xy$, which implies $d(G_{sd} xy) \geq 3$. Since $P_3: xwy$ is a subgraph of G , by R_1 , we know that $d(G_{sd} xy) \leq 3$. This implies $d(G_{sd} xy) = 3$. This is true for every $e \in E(G)$, that is G is dss.

Example

Figure 4. $d(G_1) = d(G_{1sd} xy) = 3$, $xy \in E(G_1)$ and $d(G_2) = d(G_{2sd} xy) = 3$, $xy \in E(G_2)$. Here $d(G) = d(G_{sd} ab) = 3$, for all $ab \in E(G)$.

Converse of Theorem 11 need not be true.

Example

Figure 5. G is a dss graph, $d(G) = d(G_1) = d(G_2) = 3$, $d(G_1 - \{3\}) = d(G_2 - \{6\}) = 2$, that is $d(G_1) \neq d(G_1 - \{3\})$ and $d(G_2) \neq d(G_2 - \{6\})$.

Theorem 12

If G_1 and G_2 are dss graphs such that $d(G_1) = d(G_2) = 2$, $d(G_1) = d(G_1 - \{u\})$, $d(G_2) = d(G_2 - \{v\})$, for some $u \in V_1$, $v \in V_2$, then the graph G obtained by merging u and v is also dss.

Proof

By Theorem 10, we know that $d(G) = 2$. For any graph G ,

$d(G) \geq 2$, which implies $d(G_{sd} xy) \geq 2$, for all $xy \in E(G)$. For any $e \in E(G)$, either $e \in E(G_1)$ or $e \in E(G_2)$. Assume that $e \in E(G_1)$. If possible assume that $d(G_{sd} xy) = 3$, $xy \in E(G_1)$. By R_3 , there is a domatic partition $d(G) = |\{V_1\}, \{V_2\}|$, $x \in V_1$, $y \in V_2$ such that

- i. x is 2 – dominated with respect to V_1 .
- ii. $V_2 = V_{12} \cup V_{22}$, where
 - a. $y \in V_{12}$, y is 2 – dominated with respect to V_{12} .
 - b. V_{22} does not dominate atleast one of x or y , that is V_{22} is a dominating set for $G - \{x\} - \{y\}$. $\{V_1, V_{12}, V_{22} \cup \{w\}\}$ is a domatic partition for $G_{1sd} xy$ such that $x \in V_1$, $y \in V_{12}$, which implies $d(G_{1sd} xy) = 3$, a contradiction as G_1 is dss. This implies $d(G_{sd} xy) \neq 3$, that is $d(G_{sd} xy) = 2$. Hence G is dss.

Example

Figure 6. $d(G_1) = d(G_{1sd} xy) = 2$, $xy \in E(G_1)$ and $d(G_2) = d(G_{2sd} xy) = 2$, $xy \in E(G_2)$. Here $d(G) = d(G_{sd} ab) = 2$, for all $ab \in E(G)$.

Conclusion

Using R_1 , for the graph G , G_1 , G_2 in the above notations of Theorem 11 and 12, we conclude that, if G_1 and G_2 are dss, then G is also dss.

Vestergaard, P. D and Zelinka. B. [2], have proved the following results.

Let H be the graph obtained from two disjoint graphs H_1 , H_2 by joining a vertex a_1 of H_1 with a vertex a_2 of H_2 by a bridge b . By H_1' we denote the graph obtained from H_1 by deleting a_1 , by H_2' the graph obtained from H_2 by deleting a_2 .

R_5 . For the domatic numbers of H , H_1 , H_2 the following inequalities hold:

$$\min \{d(H_1), d(H_2)\} \leq d(H) \leq 1 + \min \{d(H_1), d(H_2)\}.$$

R_6 . For the graphs H , H_1 , H_2 in the above notations the equality $d(H) = 1 + \min \{d(H_1), d(H_2)\}$ holds if and only if the following condition is fulfilled: For each $i \in \{1, 2\}$ such that $d(H_i) = \min \{d(H_1), d(H_2)\}$ there exists a partition $\{D_1^i, D_2^i, \dots, D_{d+1}^i\}$ (where $d = d(H_i)$) of the vertex set of H_i such that $D_1^i, \dots, D_d^i$ are dominating sets in H_i and D_{d+1}^i is a dominating set in H_i' , but not in H_i .

N: Let G_1 and G_2 be two graphs such that $d(G_1) = d(G_2)$

$= 3$. Let $u \in V(G_1)$, $v \in V(G_2)$. Let G be the graph obtained by joining u and v with a bridge.

Theorem 13

Let G , G_1 , G_2 be the graphs satisfying the above notation N. Then $d(G) = 3$ and G is dss if and only if there exist no partition $D_1^i, D_2^i, D_3^i, D_4^i$ for the vertex set of G_i such that D_1^i, D_2^i, D_3^i are dominating sets for G_i , D_4^i are dominating sets for G_i' but not for G_i .

Proof

By R_5 and R_6 $d(G) \leq 3$. Since $d(G_1) = d(G_{1sd} xy) = d(G_2) = d(G_{2sd} xy) = 3$, let $d(G_1) = |\{U_1\}, \{U_2\}, \{U_3\}|$, $d(G_{1sd} xy) = |\{X_1\}, \{X_2\}, \{X_3\}|$, $d(G_2) = |\{V_1\}, \{V_2\}, \{V_3\}|$, $d(G_{2sd} xy) = |\{Y_1\}, \{Y_2\}, \{Y_3\}|$. $|U_i \cup V_i|$, $i=1, 2, 3$ is a domatic partition for G , which implies $d(G) = 3$.

Let $e \in E(G)$, either $e \in E(G_1)$ or $e \in E(G_2)$ or $e = uv$.

- i. If $e = xy \in E(G_1)$, then $|X_i \cup V_i|$ is a domatic partition for G , $i = 1, 2, 3$.
- ii. If $e = xy \in E(G_2)$, then $|Y_i \cup U_i|$ is a domatic partition for G , $i = 1, 2, 3$.
- iii. If $e = uv$, let us assume that $u \in U_1$, $v \in V_1$. $U_1 \cup V_2$ is a dominating set for $G_{sd} uv$, since v is 2 – dominated. $U_2 \cup V_1$ is a dominating set for $G_{sd} uv$, since u is 2 – dominated. $U_3 \cup V_3 \cup \{w\}$ is a dominating set for $G_{sd} uv$, that is $d(G_{sd} uv) = |\{U_1 \cup V_2\}, \{U_2 \cup V_1\}, \{U_3 \cup V_3 \cup \{w\}\}|$ such that $u \in U_1 \cup V_2$, $v \in U_2 \cup V_1$.

From i, ii and iii we conclude that $d(G) \geq 3$. Since $P_3: xwy$ is a subgraph of G , by R_1 , we know that $d(G_{sd} xy) \leq 3$, which implies $d(G_{sd} xy) = 3$.

We conclude that $d(G) = d(G_{sd} ab) = 3$, for all $ab \in E(G)$.

Assume that, G is a dss graph such that $d(G) = 3$. By our assumption $d(G_1) = d(G_2) = 3$, G_1 and G_2 are dss graphs. Let $G_1' = G_1 - \{u\}$, $G_2' = G_2 - \{v\}$. If there is a partition $d(G_1) = |\{D_1^i\}, \{D_2^i\}, \{D_3^i\}, \{D_4^i\}|$, so that D_1^i, D_2^i, D_3^i are dominating sets for G_i , D_4^i are dominating sets for G_i' but not for G_i , then by R_6 $d(G) = 4$, a contradiction to our assumption $d(G) = 3$. This implies G , G_1 , G_2 are graphs which satisfy the condition of the theorem.

Example

Figure 7. $d(G_1) = d(G_{1sd} xy) = 3$, $xy \in E(G_1)$ and $d(G_2) = d(G_{2sd} xy) = 3$, $xy \in E(G_2)$. Here $d(G) = d(G_{sd} ab) = 3$, for all $ab \in E(G)$.

Theorem 14

With the notation for the domatic numbers of G , G_1 , G_2 , d

$d(G) = 2$ and G is dss.

Proof

For any $e \in E(G)$, either $e \in E(G_1)$ or $e \in E(G_2)$ or $e = uv$. If possible assume that $d(G_{sd}xy) = 3$, $x \perp y$.

i. $e = xy \in E(G_1)$.

Proof is similar to the proof of Theorem 12. As in Theorem 12, we get a contradiction, since $\{V_1, V_{12}, V_{22} \cup \{w\}\}$ is a domatic partition for $G_{1sd}xy$.

ii. $e = xy \in E(G_2)$.

Contradiction as in (i).

iii. $e = uv$

Let $d(G) = |\{V_1\}, \{V_2\}|$, that satisfies the conditions of R_3 . Let $V_1 = X_1 \cup X_2$, where $V(X_1) \subseteq V(G_1)$, $V(X_2) \subseteq V(G_2)$. Let $V_{12} = Y_1 \cup Y_2$, where $V(Y_1) \subseteq V(G_1)$, $V(Y_2) \subseteq V(G_2)$. $V_{22} = Z_1 \cup Z_2$, where $V(Z_1) \subseteq V(G_1)$, $V(Z_2) \subseteq V(G_2)$. Let $Z = \{X_1, Y_1, Z_1\}$. Z is a domatic partition for G_1 such that X_1, Y_1 are dominating sets for G_1 , Z_1 dominates $G_1 - \{u\}$, $d(G) = 1 + \min\{d(G_1), d(G_2)\} = 1 + 2 = 3$, a contradiction as $d(G) = 2$.

From i, ii and iii $d(G_{sd}uv) \neq 3$, that is $d(G_{sd}uv) = 2$. Hence G is dss.

Example

Figure 8. $d(G_1) = d(G_{1sd}xy) = 2$, $xy \in E(G_1)$ and $d(G_2) = d(G_{2sd}xy) = 2$, $xy \in E(G_2)$. Here $d(G) = d(G_{sd}ab) = 2$, for all $ab \in E(G)$.

Conclusion

For the graph G, G_1, G_2 in the above notations of theorem 13 and 14, we conclude that, if G_1 and G_2 are dss, then G is also dss.

4. Conclusion and Future Work

We have used only two graph operations for generating new dss from existing ones. One can generate new dss graphs by applying other graph operations like union, intersection, ring sum etc. Just like the operation to build the larger graphs from smaller ones, one can also do the opposite, decompose a large graph, that is dss graph to smaller graphs that are dss.

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