Universal Journal of Applied Mathematics 2(3): 136-140, 2014 http://www.hrpub.org
DOI: 10.13189/ujam.2014.020303

Navier –Stokes Second Exact Transformation

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Abstract In this article second Navier – Stokes (NSE) exact transformation to the simpler equations is covered. This transformation is executed by classical methods of Mathematical Analysis. It is shown that 3-D NSE can be conversed to a traditional vector form which looks like 2-D Vorticity Transport Equations. Second result, as well as first one, is very important for the solution of Navier–Stokes existence and smoothness one of seven Millennium Prize Problems that were stated by the Clay Mathematics Institute. The proof of solution' existence of such equations is simpler than traditional NSE. New equations will simplify the solutions of many other problems of Applied Mathematics in engineering, aeronautics, etc.

Keywords Incompressible Fluid, Navier – Stokes Equations, Vorticity Transport Equations, Vector –Valued Function, Acceleration Vector, Chain Rule, Pseudovector, Antisymmetric Tensor, Millennium Prize Problems

1. Introduction

1.1. Article's Aim

In previous article [1] first Navier – Stokes (NSE) exact transformations to the simpler equivalent equations was covered. The purpose of preparing this article is to prove that the 3-D NSE exact transformation to other simpler equations is possible by another way. This second result, as well as first one, should facilitate the solution of Navier–Stokes existence and smoothness one of seven Millennium Prize Problems that were stated by the Clay Mathematics Institute. Also, these new equations will simplify the solutions of many other problems of Applied Mathematics in engineering, atmospheric sciences, aeronautics, etc. Other 3-D NSE exact transformations to the simpler equivalent equations are not known.

1.2. General Data

The equations of motion of viscous incompressible fluid also called the Navier–Stokes equations (NSE) together with the well known continuity equation can be written as follows from [2, p. 174]

$$\rho\vec{F} - \operatorname{grad} p + \mu\nabla^2\dot{\vec{u}} = \rho\ddot{\vec{u}}\ , \tag{1}$$

$$\operatorname{div}\dot{\vec{u}} = \frac{\partial \dot{u}_x}{\partial x} + \frac{\partial \dot{u}_y}{\partial y} + \frac{\partial \dot{u}_z}{\partial z} = 0\ . \tag{2}$$

Here, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots$ - vectors sum of a given, externally applied forces (e.g. gravity $\vec{F}_1$, magnetic $\vec{F}_2$ and other), p - pressure, $\dot{\vec{u}}$ - velocity vector, $\ddot{\vec{u}} = d\dot{\vec{u}}/dt$ - acceleration vector, ρ - density, μ - viscosity, ∇^2 - Laplace operator.

Note that pressure p can be eliminated by taking the operator rot (also known curl) of both sides of (1). This well-known approach belongs to Helmholtz. As a result of such transformations the Vorticity Transport Equations (VTE) were obtained [3, p. 294, 321; 4, p. 74]. In two dimensions (2-D), it is well known that (if $\vec{F} = 0$)

$$\nu\nabla^2\Omega = \frac{d\Omega}{dt},\quad 2\Omega = \operatorname{rot}\dot{\vec{u}},\quad \nu = \frac{\mu}{\rho}. \tag{3}$$

In three dimensions (3-D) it is known for a long time that VTE contain the additional terms [3, p. 294]. Therefore 3-D VTE are very difficult for solutions of important mathematical problems in engineering. Standard methods from PDE appear inadequate to solve these equations. After all this negativity let's restrict attention to any positive suggestions for how to solve this unpleasant problem. The analysis of the additional terms shows that 3-D VTE can be transformed. After transformation the vector form of 3-D VTE look like (3).

2. Method of 3-D VTE Transformation

2.1. Analysis of 3-D VTE Additional Terms

For visibility let's consider the well known NSE transformation. If we apply the operator rot formula (1) becomes

$$\operatorname{rot}\vec{F} + \nu\nabla^2\operatorname{rot}\dot{\vec{u}} = \operatorname{rot}\ddot{\vec{u}}. \tag{1*}$$

To do the analysis of the additional terms referenced above we should write the expressions for components $\operatorname{rot}_i \ddot{\vec{u}}$ (i. e. right - hand side of above equation). The well-known form of these components in Cartesian coordinates can be written so

$$\operatorname{rot}_x \ddot{\vec{u}} = \frac{\partial \ddot{u}_z}{\partial y} - \frac{\partial \ddot{u}_y}{\partial z}, \quad \operatorname{rot}_y \ddot{\vec{u}} = \frac{\partial \ddot{u}_x}{\partial z} - \frac{\partial \ddot{u}_z}{\partial x}, \quad \operatorname{rot}_z \ddot{\vec{u}} = \frac{\partial \ddot{u}_y}{\partial x} - \frac{\partial \ddot{u}_x}{\partial y} \tag{4}$$

Then let's consider the derivation of expanded expression for any one component (for example $\operatorname{rot}_x \ddot{\vec{u}}$). The acceleration vector components $\ddot{u}_i$ can be written following [2, p. 39]

$$\ddot{u}_x = \frac{d\dot{u}_x}{dt} = \frac{\partial \dot{u}_x}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_x}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_x}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_x}{\partial z},$$
$$\ddot{u}_y = \frac{d\dot{u}_y}{dt} = \frac{\partial \dot{u}_y}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_y}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_y}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_y}{\partial z},$$
$$\ddot{u}_z = \frac{d\dot{u}_z}{dt} = \frac{\partial \dot{u}_z}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_z}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_z}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_z}{\partial z}.$$

Let's differentiate the expressions of acceleration components to determine $\operatorname{rot}_x \ddot{\vec{u}}$ according (4)

$$\frac{\partial \ddot{u}_z}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial \dot{u}_z}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_z}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_z}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_z}{\partial z} \right) = \frac{\partial^2 \dot{u}_z}{\partial y \partial t} + \dot{u}_x \frac{\partial^2 \dot{u}_z}{\partial y \partial x} + \frac{\partial \dot{u}_x}{\partial y}\frac{\partial \dot{u}_z}{\partial x} + \dot{u}_y \frac{\partial^2 \dot{u}_z}{\partial y^2} + \frac{\partial \dot{u}_y}{\partial y}\frac{\partial \dot{u}_z}{\partial y} + \dot{u}_z \frac{\partial^2 \dot{u}_z}{\partial y \partial z} + \frac{\partial \dot{u}_z}{\partial y}\frac{\partial \dot{u}_z}{\partial z},$$

$$\frac{\partial \ddot{u}_y}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\partial \dot{u}_y}{\partial t} + \dot{u}_x \frac{\partial \dot{u}_y}{\partial x} + \dot{u}_y \frac{\partial \dot{u}_y}{\partial y} + \dot{u}_z \frac{\partial \dot{u}_y}{\partial z} \right) = \frac{\partial^2 \dot{u}_y}{\partial z \partial t} + \dot{u}_x \frac{\partial^2 \dot{u}_y}{\partial z \partial x} + \frac{\partial \dot{u}_x}{\partial z}\frac{\partial \dot{u}_y}{\partial x} + \dot{u}_y \frac{\partial^2 \dot{u}_y}{\partial z \partial y} + \frac{\partial \dot{u}_y}{\partial z}\frac{\partial \dot{u}_y}{\partial y} + \dot{u}_z \frac{\partial^2 \dot{u}_y}{\partial z^2} + \frac{\partial \dot{u}_z}{\partial z}\frac{\partial \dot{u}_y}{\partial z}.$$

After transformation of $\frac{\partial \ddot{u}_z}{\partial y} - \frac{\partial \ddot{u}_y}{\partial z}$ we obtain the expression $\operatorname{rot}_x \ddot{\vec{u}}$

$$\frac{1}{2}\operatorname{rot}_x \ddot{\vec{u}} = \bar{\Omega}_x = \frac{\partial \Omega_x}{\partial t} + \dot{u}_x \frac{\partial \Omega_x}{\partial x} + \dot{u}_y \frac{\partial \Omega_x}{\partial y} + \dot{u}_z \frac{\partial \Omega_x}{\partial z} + \Omega_x \frac{\partial \dot{u}_y}{\partial y} + \Omega_x \frac{\partial \dot{u}_z}{\partial z} + \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial y}\frac{\partial \dot{u}_z}{\partial x} - \frac{\partial \dot{u}_x}{\partial z}\frac{\partial \dot{u}_y}{\partial x} \right) = \frac{d\Omega_i}{dt} + \Omega_x \frac{\partial \dot{u}_y}{\partial y} + \Omega_x \frac{\partial \dot{u}_z}{\partial z} + \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial y}\frac{\partial \dot{u}_z}{\partial x} - \frac{\partial \dot{u}_x}{\partial z}\frac{\partial \dot{u}_y}{\partial x} \right) \tag{5}$$

The expressions for $\operatorname{rot}_y \ddot{\vec{u}}$, $\operatorname{rot}_z \ddot{\vec{u}}$ can be written by analogy.

Now let's consider well-known traditional transformation of (5). For 2-D flow $\dot{u}_x = 0$. Therefore from (5) we find

$$\frac{1}{2}\operatorname{rot}_i \ddot{\vec{u}} = \bar{\Omega}_i = \frac{d\Omega_i}{dt} + \Omega_i \operatorname{div}\dot{\vec{u}}. \tag{6}$$

For 3-D flow two terms in brackets of (5) probably are $\Omega_x \frac{\partial \dot{u}_x}{\partial x}$ and the last three terms give $\Omega_x \operatorname{div}\dot{\vec{u}}$ (by analogy). However, it is only our supposition. This supposition will be proven next.

2.2. Proof of Supposition (6) for 3-D VTE

First note that any vector on Euclidean space $\vec{u} = \vec{u}(x, y, z)$ can be represented as $\vec{u} = \vec{u}(\varsigma)$, $\varsigma = \varsigma(x, y, z)$. This representation is well known as a vector function of scalar argument (also called vector-valued function) [5, p. 514]. Therefore the velocity vector $\dot{\vec{u}} = \dot{\vec{u}}(x, y, z, t)$ for a fixed time $t = \bar{t}$ can be represented as $\dot{\vec{u}} = \dot{\vec{u}}(\varsigma)$, $\varsigma = \varsigma(x, y, z)$. Then using formula (1) in [5, p. 644] we obtain

$$\frac{\partial \dot{\vec{u}}}{\partial x_i} = \frac{\partial \dot{\vec{u}}}{\partial \varsigma}\frac{\partial \varsigma}{\partial x_i}, (x_i = x, y, z) . \tag{7}$$

Note that formulas (7) also known as chain rule [3, p. 77]. After transformations of formulas (7) we obtain such equalities (a complete transformation can be found in [1])

$$\frac{\partial \dot{u}_x}{\partial x_i}\frac{\partial \dot{u}_y}{\partial x_j} = \frac{\partial \dot{u}_x}{\partial x_j}\frac{\partial \dot{u}_y}{\partial x_i}, \quad \frac{\partial \dot{u}_x}{\partial x_i}\frac{\partial \dot{u}_z}{\partial x_j} = \frac{\partial \dot{u}_x}{\partial x_j}\frac{\partial \dot{u}_z}{\partial x_i}, \quad \frac{\partial \dot{u}_y}{\partial x_i}\frac{\partial \dot{u}_z}{\partial x_j} = \frac{\partial \dot{u}_y}{\partial x_j}\frac{\partial \dot{u}_z}{\partial x_i} \tag{8}$$

Now we will need to check a possibility of this supposition for formula (5)

$$\frac{\partial \dot{u}_x}{\partial y}\frac{\partial \dot{u}_z}{\partial x} - \frac{\partial \dot{u}_x}{\partial z}\frac{\partial \dot{u}_y}{\partial x} = \Omega_x \frac{\partial \dot{u}_x}{\partial x} . \tag{9}$$

For visibility of our transformation it is useful to write together relations (8) and any equalities (implying from them and required for following transformations)

$$\frac{\partial \dot{u}_x}{\partial x_i}\frac{\partial \dot{u}_y}{\partial x_j}=\frac{\partial \dot{u}_x}{\partial x_j}\frac{\partial \dot{u}_y}{\partial x_i}\Rightarrow \frac{\partial \dot{u}_x}{\partial z}\frac{\partial \dot{u}_y}{\partial x}=\frac{\partial \dot{u}_x}{\partial x}\frac{\partial \dot{u}_y}{\partial z},$$
$$\frac{\partial \dot{u}_x}{\partial x_i}\frac{\partial \dot{u}_z}{\partial x_j}=\frac{\partial \dot{u}_x}{\partial x_j}\frac{\partial \dot{u}_z}{\partial x_i}\Rightarrow \frac{\partial \dot{u}_x}{\partial y}\frac{\partial \dot{u}_z}{\partial x}=\frac{\partial \dot{u}_x}{\partial x}\frac{\partial \dot{u}_z}{\partial y}, \quad (10)$$
$$\frac{\partial \dot{u}_y}{\partial x_i}\frac{\partial \dot{u}_z}{\partial x_j}=\frac{\partial \dot{u}_y}{\partial x_j}\frac{\partial \dot{u}_z}{\partial x_i}.$$

The equalities after $\Rightarrow$ confirm the validity of formulas (6) for 3-D flow because formula (5) becomes

$$\frac{1}{2}\mathrm{rot}_x\,\ddot{\vec{u}}=\bar{\Omega}_x=\frac{\partial\Omega_x}{\partial t}+\dot{u}_x\frac{\partial\Omega_x}{\partial x}+\dot{u}_y\frac{\partial\Omega_x}{\partial y}+\dot{u}_z\frac{\partial\Omega_x}{\partial z}+$$
$$+\Omega_x\left(\frac{\partial \dot{u}_x}{\partial x}+\frac{\partial \dot{u}_y}{\partial y}+\frac{\partial \dot{u}_z}{\partial z}\right). \quad (11)$$

As we can see equation (11) coincides with equation (6).

The expressions for two other components can be written by analogy. As mentioned above for incompressible fluid $\mathrm{div}\,\vec{u}=0$. Therefore 3-D VTE look like (3) if $\vec{F}=0$. In component form we obtain three equations (if $\vec{F}\neq 0$)

$$\frac{1}{2}\mathrm{rot}_i\,\vec{F}+\nu\nabla^2\Omega_i=\frac{d\Omega_i}{dt},\ \ (i=x,y,z)\,. \quad (3^*)$$

Thus we can see that equations (3*) can be derived as the 3-D Navier–Stokes exact transformation if we apply the operator rot (curl) and additional equalities (10).

3. Discussion of main results

3.1. VTE Contradiction

Three equations (3*) can be represented as one vector equation with one variable $\Omega=(\mathrm{rot}\,\dot{\vec{u}})/2$

$$\frac{1}{2}\mathrm{rot}\,\vec{F}+\nu\nabla^2\Omega=\frac{d\Omega}{dt}. \quad (3^{**})$$

In mathematics and physics the rot (curl) is an operation which takes any vector field **A** and produces another vector field rot **A**. However it is known that rot **A** (also called pseudovector) is equivalent to an antisymmetric tensor [2, p. 183; 8, p. 104]. In that case under co-ordinate change the components of the antisymmetric tensor should transform differently from the true vector components. Therefore the author of textbook [7, p. 50] paid attention that «… pseudovector … from the point of view of its vector product on other true vector is equivalent to antisymmetric tensor, but as vector cannot be equal to tensor». In other words, the antisymmetric tensor can be found as decomposition of any rank-2 tensor. For example [7, p.62, 63] a tensor of partial derivates of velocity vector can be written as a sum of symmetric and antisymmetric parts:

$$\begin{pmatrix}\frac{\partial \dot{u}_x}{\partial x} & \frac{\partial \dot{u}_x}{\partial y} & \frac{\partial \dot{u}_x}{\partial z}\\ \frac{\partial \dot{u}_y}{\partial x} & \frac{\partial \dot{u}_y}{\partial y} & \frac{\partial \dot{u}_y}{\partial z}\\ \frac{\partial \dot{u}_z}{\partial x} & \frac{\partial \dot{u}_z}{\partial y} & \frac{\partial \dot{u}_z}{\partial z}\end{pmatrix}=$$
$$\begin{pmatrix}\frac{\partial \dot{u}_x}{\partial x} & \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial y}+\frac{\partial \dot{u}_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial z}+\frac{\partial \dot{u}_z}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial \dot{u}_y}{\partial x}+\frac{\partial \dot{u}_x}{\partial y}\right) & \frac{\partial \dot{u}_y}{\partial y} & \frac{1}{2}\left(\frac{\partial \dot{u}_y}{\partial z}+\frac{\partial \dot{u}_z}{\partial y}\right)\\ \frac{1}{2}\left(\frac{\partial \dot{u}_z}{\partial x}+\frac{\partial \dot{u}_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial \dot{u}_z}{\partial y}+\frac{\partial \dot{u}_y}{\partial z}\right) & \frac{\partial \dot{u}_z}{\partial z}\end{pmatrix}+$$
$$+\begin{pmatrix}0 & \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial y}-\frac{\partial \dot{u}_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial \dot{u}_x}{\partial z}-\frac{\partial \dot{u}_z}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial \dot{u}_y}{\partial x}-\frac{\partial \dot{u}_x}{\partial y}\right) & 0 & \frac{1}{2}\left(\frac{\partial \dot{u}_y}{\partial z}-\frac{\partial \dot{u}_z}{\partial y}\right)\\ \frac{1}{2}\left(\frac{\partial \dot{u}_z}{\partial x}-\frac{\partial \dot{u}_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial \dot{u}_z}{\partial y}-\frac{\partial \dot{u}_y}{\partial z}\right) & 0\end{pmatrix}$$

As we can see the symmetric part is a velocity deformation tensor $(\dot{\varepsilon}_{ij})$, $\dot{\varepsilon}_{ij}=\dot{\varepsilon}_{ji}$. The antisymmetric part is antisymmetric tensor which contains six components $\mp\mathrm{rot}_i\,\dot{\vec{u}}$ (also called pseudovector components). Therefore the above expression can be written so

$$\begin{pmatrix}\frac{\partial \dot{u}_x}{\partial x} & \frac{\partial \dot{u}_x}{\partial y} & \frac{\partial \dot{u}_x}{\partial z}\\ \frac{\partial \dot{u}_y}{\partial x} & \frac{\partial \dot{u}_y}{\partial y} & \frac{\partial \dot{u}_y}{\partial z}\\ \frac{\partial \dot{u}_z}{\partial x} & \frac{\partial \dot{u}_z}{\partial y} & \frac{\partial \dot{u}_z}{\partial z}\end{pmatrix}=\begin{pmatrix}\dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz}\\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy} & \dot{\varepsilon}_{yz}\\ \dot{\varepsilon}_{zx} & \dot{\varepsilon}_{zy} & \dot{\varepsilon}_{zz}\end{pmatrix}+$$
$$+\begin{pmatrix}0 & -\frac{1}{2}\mathrm{rot}_z\,\dot{\vec{u}} & \frac{1}{2}\mathrm{rot}_y\,\dot{\vec{u}}\\ \frac{1}{2}\mathrm{rot}_z\,\dot{\vec{u}} & 0 & -\frac{1}{2}\mathrm{rot}_x\,\dot{\vec{u}}\\ -\frac{1}{2}\mathrm{rot}_y\,\dot{\vec{u}} & \frac{1}{2}\mathrm{rot}_x\,\dot{\vec{u}} & 0\end{pmatrix}. \quad (12)$$

The above results show very well that new variable $\Omega=(\mathrm{rot}\,\dot{\vec{u}})/2$ in the transformed NSE is not a true vector function. Therefore the above definition of (3**) as "vector equation" is incorrect and must be reconsidered (for example, "equation in vector form"). A complete analysis of this important problem and other contradictions in different textbooks [2, 3, 4, 7] we will consider in another paper.

3.2. Confirming of Main Result by Analogy

As we can see 1-D, 2-D and 3-D NSE in the vector form are identical. If we apply the operator rot these equations are identical also. This conclusion follows from (1*). In that case, by analogy, 1-D, 2-D and 3-D VTE must be identical too. After compare of equations (3), (3*) and (3**) we can see that 3-D VTE in the vector form look like 2-D VTE. It is an additional confirmation that our mathematical proof is correct,

3.3. Applications of Main Result

A complete description of this problem needs carefully discussion. We can see that equations (3**) are simpler than system of equations (1), (2). Each of three equations (3*) include only one variable $\Omega_i = \mathrm{rot}_i\, \dot{\vec{u}}/2$ because the pressure p can be deleted if we apply the operator rot (curl). In two dimensions, this result is well known for a long time from textbooks [3, p. 321; 4, p. 74]. However for the proof of 2-D problem Navier–Stokes existence and smoothness this result for some reason has not been discussed [6]. But, this result is used for the solutions of applied mathematical problems in engineering [9].

This second exact transformation, as well as first one [1], should facilitate the solution of the Navier–Stokes Millennium Prize Problem. However, some authors claim that this problem even may be solved by transforming the NSE into an equivalent system. A complete description of this and other approaches can be found here [10]. The author of this cited work claims: “Thus, one is left with only three possible strategies if one wants to solve the full problem”. The formulation of “strategy 1” look like “solve the Navier-Stokes equation exactly and explicitly (or at least transform this equation exactly and explicitly to a simpler equation)”.

4. Conclusion

As follows from the different books [2, p. 294, 321; 3, p. 74] NSE (1) can be conversed to the VTE by taking the operator rot (also known curl). However the well-known vector forms of the 3-D and 2-D VTE are different. In this paper we have shown that 3-D equations in the vector form can be conversed and look like 2-D equations. As we can see the conversation method is based on an exact transformation of formulas (7) also known as chain rule. In 3.2. we can see the confirming of this result by analogy. It is an additional confirmation that our mathematical proof is correct,

This exact NSE transformation, as well as first one [1], should facilitate the solution of the Navier–Stokes Millennium Prize Problem. All proofs of the Navier–Stokes existence and smoothness should use the additional conditions (8). However, some authors (for example [10]) even claim that the Navier–Stokes Millennium Prize Problem may be solved only by transforming the NSE into an equivalent system. Other 3-D NSE exact transformations to the simpler equivalent equations are not known.

From new basic equations (3*) or (3**) one can go to solve more complicated problems of applied mathematics in engineering, atmospheric sciences, aeronautics, etc. Equations (3**) and (11) in [1] can be used for the checking of so called NSE exact solutions. All true exact solutions of the NSE should satisfy these equations. Otherwise such “NSE exact solutions” are false.

The next unpleasant things we have also established for such well-known classical rule of operation rot (curl). After comparison of information in different textbooks [2, 7] we have established that new variable $\Omega = (\mathrm{rot}\, \dot{\vec{u}})/2$ in the transformed NSE (3**) is not a true vector function. It is known [2, p. 183; 8, p. 104] that components of Ω (sometime called pseudovector) is equivalent to the antisymmetric tensor components. The additional confirmation of this claim we can obtain after analysis of formula (12). Hence this claim probably is correct. The consideration of this important problem we prolong in another paper.

Acknowledgements

The author thanks three anonymous referees for many useful remarks and comments. These remarks have helped to improve the submitted manuscript and to prepare this new version.

REFERENCES

[1] Alexandr Kozachok. Navier –Stokes First Exact Transformation, Universal Journal of Applied Mathematics, 1, 157 - 159. doi: 10.13189/ujam.2013.010301, 2013 (English) http://www.hrpub.org/download/20131107/UJAM1-12600416.pdf

[2] L.I. Sedov. Mechanics of Continuous Media, v. 1. Textbook, Science, Moscow, 1970http://eqworld.ipmnet.ru/ru/library/books/Sedov_MSS_t1_1970ru.djvu) . (Russian) ISBN-13: 978-9971507282 (English)

[3] J.H. Heinbockel. Introduction to Tensor Calculus and Continuum Mechanics., Trafford Publishing, 2001, ISBN-13: 978-1553691334 (English) http://lib.prometey.org/?id=15227

[4] G.Shlihting. The theory of a boundary layer, Science, Moscow, 1969(Russian)http://eqworld.ipmnet.ru/ru/library/books/Schlichting1974ru.djvu

[5] M.J. Vygodsky. Manual on Higher Mathematics, (12th edition). Science, 1977 (Russian)http://eqworld.ipmnet.ru/ru/library/books/Vygodskij1977ru.djvu . ASIN: B001U5VF9O (English)

[6] O. Ladyzhenskaya. The Mathematical Theory of Viscous Incompressible Flows (2nd edition), Gordon and Breach,

New York, 1969. (English)

[7] L.G.Lojtsjansky. Fluid and Gas Mechanics, Manual, Science, Moscow, 1970(Russian)http://eqworld.ipmnet.ru/ru/library/books/Lojcyanskij1950ru.djvu

[8] A.I. Borisenko, I.E. Tarapov. Vector Analysis and beginnings of Tensors Calculus (3th edition), Manual, Higher school, Moscow, 1966 (Russian) http://eqworld.ipmnet.ru/ru/library/books/BorisenkoTarapov1966ru.djvu . ISBN-13: 978-0486638331(English)

[9] Felix Kaplanski. The vorticity equation and its application. Tallin University of Technology. (English) http://www.brighton.ac.uk/shrl/shrl/events/The%20vorticity%20equation%20and%20its%20applications.ppt

[10] Terence Tao. Why global regularity for Navier-Stokes is hard (research and expository papers, discussion of open problems, and other maths-related topics), 2007 http://terrytao.wordpress.com/2007/03/18/why-global-regularity-for-navier-stokes-is-hard/