MHD Flow though Rotating Porous Medium with Radiating Heat Transfer in the Presence of Fluctuating Thermal Diffusion

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Abstract This communication investigates the effect of magnetic field and radiating heat transfer on unsteady free convection viscous incompressible electric conducting fluid past a vertical surface in a rotating porous medium. It is assumed that surface is rotating with angular velocity \( \Omega \) and the porous vertical surface is subjected to constant suction velocity. The variable heat flux is also assumed on the vertical surface varies with time; the governing equations are solved by adopting complex variable notations. The analytical expressions for velocity and temperature fields are obtained. The effects of various parameters on mean velocity, mean temperature, transient velocity and transient temperature have been discussed and shown graphically.

Keywords MHD, Porous Medium, Incompressible Fluid, Heat Flux, Radiating Heat Transfer

In view of geophysical applications of the flow through porous medium, a series of investigations has been made by Raptis et al. [5-7], where the porous medium is either bounded by horizontal, vertical surfaces or parallel porous plates. Singh et al. [8] and Lai and Kulacki [9] have been studied the free convective flow past vertical wall. Nield [10] studied convection flow through porous medium with inclined temperature gradient. Singh et al. [11] also studied periodic solution on oscillatory flow through channel in rotating porous medium. Further due to increasing scientific and technical applications on the effect of radiation on flow characteristic has more importance in many engineering processes occurs at very high temperature and acknowledge radiative heat transfer such as nuclear power plant, gas turbine and various propulsion devices for aircraft, missile and space vehicles. The effect of radiation on flow past different geometry a series of investigation have been made by Hassan [12], Seddeek [13] and Sharma et al [14].

Therefore the object of the present paper is to investigate the effect of magnetic field and radiation on unsteady free convection flow past a porous vertical surface in a rotating porous medium. Assuming periodic thermal diffusion at the plate, the analytical solution is obtained using regular perturbation technique and discussed graphically.

1. Introduction

The flow problems of an electrically conducting fluid under the influence of magnetic field have attracted the interest of many authors in view of its applications to geophysics, astrophysics, and in the field of aerodynamics. In view of the increasing technical applications using Magnetohydrodynamics effect, it is desirable to extend many of the available viscous hydrodynamic solution to include the effects of magnetic field for those cases when the viscous fluid is electrically conducting. Rossow [1], Greenspan and Carrier [2] have studied extensively the hydromagnetic effects on the flow past a plate with or without injection/suction. The hydromagnetic channel flow and temperature field was investigated by Attia and Kotab [3]. Hossain et al. [4] have studied the MHD free convection flow when the surface kept at oscillating surface heat flux. The buoyancy-induced flows of electrically conducting fluid through saturated porous media have been a prime topic of many studied during the past several years.

2. Formulation of the Problem

We consider the unsteady viscous incompressible electrically conducting fluid through a porous medium, occupying a semi-infinite region of the space bounded by a vertical infinite porous region of the space bounded by a vertical infinite porous region in a rotating system, when the temperature of the surface, varies with time. We assume the effect of radiation on vertical surface which is subjected to uniform constant suction velocity in the direction perpendicular to surface. We consider the vertical infinite porous surface rotating with constant angular velocity \( \Omega \) about an axis which is perpendicular to the vertical plane confined with a viscous fluid occupying the porous region.
Vertical porous plane is taken to be \( z^* = 0 \) plane with \( z^* \) axis normal to it. \( X^* \) axis is selected vertically upwards and \( y^* \) axis in the perpendicular direction in \( z^* = 0 \) plane. A magnetic field of constant intensity is applied perpendicular to the vertical surface. The flow is assumed to be along the plane \( z^* = 0 \). With the above frame of reference and assumptions, with physical variables, except the pressure \( p \), are the function of \( z^* \) and \( t^* \) only. The flow in porous medium involves small velocities permitting the neglect of heat due to viscous dissipation in governing equation. The equation expressing the conservation of mass and energy transfer in rotating frame of reference are given by

\[
\frac{\partial w^*}{\partial z^*} = 0 , \quad \ldots (1)
\]

\[
\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2 \mu^* v^* = g \beta (T^* - T^*_\infty) + \nu \frac{\partial^2 u^*}{\partial z^*^2} - \frac{\nu u^*}{k^*} - \frac{(\vec{J} \times \vec{B})}{\rho} , \quad \ldots (2)
\]

\[
\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2 \Omega u^* = \nu \frac{\partial^2 v^*}{\partial z^*^2} - \frac{\nu v^*}{k^*} - \frac{(\vec{J} \times \vec{B})}{\rho} , \quad \ldots (3)
\]

\[
0 = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} - \frac{\nu w^*}{k^*} - \frac{(\vec{J} \times \vec{B})}{\rho} , \quad \ldots (4)
\]

where the fourth term on the right hand side of equations (2-4) is the Lorentz force due to magnetic field \( \vec{B} \), and is given by

\[
\vec{J} \times \vec{B} = \sigma (\nu \vec{B} \times \vec{B}) \times \vec{B} \quad \ldots (5)
\]

Substituting equation (5) in equations (2-4), we have

\[
\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2 \mu^* v^* = g \beta (T^* - T^*_\infty) + \nu \frac{\partial^2 u^*}{\partial z^*^2} - \frac{\nu u^*}{k^*} - \frac{\sigma B^2 u^*}{\rho} , \quad \ldots (6)
\]

\[
\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2 \mu^* u^* = \nu \frac{\partial^2 v^*}{\partial z^*^2} - \frac{\nu v^*}{k^*} - \frac{\sigma B^2 v^*}{\rho} , \quad \ldots (7)
\]

\[
0 = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} - \frac{\nu w^*}{k^*} - \frac{\sigma B^2 w^*}{\rho} , \quad \ldots (8)
\]

\[
\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \alpha \frac{\partial^2 T^*}{\partial z^*^2} - \frac{1}{\rho C_p} \frac{\partial q^*_w}{\partial z^*} , \quad \ldots (9)
\]

where \( u^*, v^*, w^* \) are components of velocity, \( g \) is the acceleration due to gravity, \( \beta \) is the volumetric coefficient of thermal expansion, \( T^* \) is the temperature, \( T^*_\infty \) is the temperature in free stream, \( \nu \) is the kinematic viscosity, \( \Omega \) is the angular velocity, \( K^* \) is the permeability, \( C_p \) is the specific heat at constant pressure, \( q^*_w \) is radiative heat flux, \( p^* \) is the pressure, \( \rho \) is the density, \( t^* \) is the time and \( \alpha \) is the thermal diffusivity.

The boundary conditions of the problem are

\[
z = 0: \ u^* = 0 , \quad v^* = 0 , \quad \frac{\partial T^*}{\partial z^*} = \frac{q^*_w}{\kappa} (1 + e^{i\omega t}) \quad \ldots (10)
\]

\[
z \to \infty : \ u^* \to 0 , \quad v^* \to 0 , \quad T^* \to T^*_\infty .
\]

where \( q^*_w \) is the heat flux at wall \( \omega^* \) is the frequency of fluctuation and \( \kappa \) is the thermal conductivity of the plate. For constant suction, we have from equation (1)

\[
w^* = - w_{0} \quad \ldots (11)
\]

Considering \( u + iv = U \) and taking into account equation (11), the equations (6) and (7) can be written as

\[
\frac{\partial U^*}{\partial t^*} - w_{0} \frac{\partial U^*}{\partial z^*} + 2i \Omega U^* = g \beta (T^* - T^*_\infty) + \nu \frac{\partial^2 U^*}{\partial z^*^2} - \frac{\nu U^*}{k^*} - \frac{\sigma B^2 U^*}{\rho} , \quad \ldots (12)
\]

We introduce the following non-dimensional quantities as:

\[
z = \frac{w_{0} z^*}{\nu} , \quad U^* = \frac{U^*}{U_{0}} , \quad t = \frac{t^* w_{0}^2}{\nu} , \quad \omega = \frac{\nu \omega^*}{w_{0}} , \quad \theta = \frac{\kappa (T^* - T^*_\infty) w_{0}}{q^*_w \nu} ,
\]

\[
\nu \frac{\partial U^*}{\partial z^*} = \frac{\partial U^*}{\partial z^*} , \quad \mu^* v^* = \nu \frac{\partial^2 v^*}{\partial z^*^2} - \frac{\nu v^*}{k^*} , \quad \ldots (1)
\]

\[
\frac{\partial U^*}{\partial t^*} - w_{0} \frac{\partial U^*}{\partial z^*} + 2i \Omega U^* = g \beta (T^* - T^*_\infty) + \nu \frac{\partial^2 U^*}{\partial z^*^2} - \frac{\nu U^*}{k^*} - \frac{\sigma B^2 U^*}{\rho} , \quad \ldots (12)
\]
Substituting these non-dimensional quantities in equations (12) and (9), we get
\[
\frac{\partial U}{\partial t} - \frac{\partial U}{\partial z} - 2 t RU = \frac{\partial^3 U}{\partial z^3} - \frac{U}{k} - M^2 U ,
\]
\[
\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^3 \theta}{\partial z^3} - F \theta ,
\]
and the boundary conditions (10) become
\[
\begin{aligned}
z & = 0: \ U = 0, \quad \frac{\partial \theta}{\partial z} = -(1 + \varepsilon \ e^{i\omega t}) \\
z \to \infty: \ U \to 0, \quad \theta \to 0 .
\end{aligned}
\]

3. Solution of the Problem

In order to solve the problem, we assume the solutions of the following form because amplitude \( \varepsilon (<< 1) \) of the variation of temperature is very small
\[
\begin{aligned}
U (z , t) & = U_o (z) + \varepsilon U_i (z) \ e^{i\omega t} + \ldots \\
\theta (z , t) & = \theta_o (z) + \varepsilon \theta_i (z) \ e^{i\omega t} + \ldots
\end{aligned}
\]
Substituting (16) in equations (13) and (14), and equating the coefficient of identical powers of \( \varepsilon \) and neglecting those of \( \varepsilon^2, \varepsilon^3 \) etc., we get
\[
\begin{aligned}
U_o + U_i - 2 t R U_o - \frac{U_o}{k} - M^2 U_o & = - Gr \theta_o , \\
U_o + U_i - 2 t R U_i - i \omega U_i - \frac{U_i}{k} - M^2 U_i & = - Gr \theta_i ,
\end{aligned}
\]
\[
\begin{aligned}
\theta_o + Pr \theta_o - F Pr \theta_o & = 0 , \\
\theta_i + Pr \theta_i - (F + i \omega ) \theta_i Pr & = 0 .
\end{aligned}
\]
The corresponding boundary conditions (15) reduce to
\[
\begin{aligned}
z & = 0: \ U_o = 0, \ U_i = 0 , \quad \frac{\partial \theta_o}{\partial z} = -1 , \quad \frac{\partial \theta_i}{\partial z} = -1 \\
z \to \infty: \ U_o \to 0 , \ U_i \to 0 , \quad \theta_o \to 0 , \quad \theta_i \to 0 .
\end{aligned}
\]
Solving equations (17) to (20) under corresponding boundary conditions (21), we get
\[
U_o(z) = c_o (e^{i\omega z} - e^{-i\omega z}) \quad \ldots \quad (22)
\]
\[
U_i(z) = c_i (e^{i\omega z} - e^{-i\omega z}) \quad \ldots \quad (23)
\]
\[ \theta_\theta (z) = \frac{1}{c_1} e^{-z c_1} \]  
\[ \theta_\phi (z) = \frac{1}{c_2} e^{-z c_2} \]  

where

\[ c_1 = \frac{1}{2} \left[ \text{Pr} + \sqrt{\text{Pr}^2 + 4 F \text{Pr}} \right] \]
\[ c_2 = \frac{1}{2} \left[ \text{Pr} + \sqrt{\text{Pr}^2 + 4 \text{Pr} (F + \phi \omega)} \right] \]
\[ c_3 = \frac{1}{2} \left[ 1 + \sqrt{1 + 8 R + \frac{4}{k} + 4 M^2} \right] \]
\[ c_4 = -\frac{\text{Gr}}{c_1 (c_1^2 - c_1 - 2 R - \frac{1}{k} - M^2)} \]
\[ c_5 = \frac{1}{2} \left[ 1 + \sqrt{1 + 4 \left( 2 R + \phi \omega \right) + \frac{4}{k} + 4 M^2} \right] \]
\[ c_6 = -\frac{\text{Gr}}{c_2 (c_2^2 - c_2 - 2 R - \phi \omega - \frac{1}{k} - M^2)} \]

4. Results

4.1. Steady flow

We take \( U_0 = u_0 + i v_0 \) in equation (22) and subsequent comparison of the real and imaginary parts gives the mean primary \( \frac{u_0}{w_0} \) and mean secondary \( \frac{v_0}{w_0} \) velocity fields as

\[ \frac{u}{w_0} = \left[ a_1 e^{-z c_1} - a_1 e^{z c_1} \cos a_1 z - a_1 e^{z c_1} \sin a_1 z \right] , \]
\[ \frac{v}{w_0} = \left[ a_1 e^{-z c_1} + a_1 e^{z c_1} \sin a_1 z - a_1 e^{z c_1} \cos a_1 z \right] \]

4.2. Unsteady flow

Replacing the unsteady parts \( U_1 (z, t) = M_1 + i \omega M_1 \) and \( \theta_1 (z, t) = T_1 + i \omega T_1 \) respectively in equation (23), we get

\[ U(z, t), \theta(z, t) = [ U_0(z), \theta_0(z) ] + \varepsilon e^{\omega t} \left[ \left( M_1 + i \omega M_1 \right), \left( T_1 + i \omega T_1 \right) \right] \]  

The primary, secondary velocity fields in terms of the fluctuating components are

\[ \frac{u}{w_0} (z, t) = u_0 + \varepsilon \left( M_1 \cos \omega t - M_1 \sin \omega t \right) \]
\[ \frac{v}{w_0} (z, t) = v_0 + \varepsilon \left( M_1 \sin \omega t + M_1 \cos \omega t \right) \]
\[ \theta (z, t) = \theta_0 + \varepsilon \left( T_1 \cos \omega t - T_1 \sin \omega t \right) \]

Taking \( \omega t = \frac{\pi}{2} \) in equations (28), (29) and (30), we get the expression for transient primary velocity, transient secondary
velocity and transient temperature as

\[
\frac{u}{w_0} \left( z, \frac{\pi}{2\omega} \right) = u_s(z) - \varepsilon \ M_1 (z), \quad \cdots (31)
\]

\[
\frac{v}{w_0} \left( z, \frac{\pi}{2\omega} \right) = v_s(z) + \varepsilon \ M_1 (z), \quad \cdots (32)
\]

\[
\theta \left( z, \frac{\pi}{2\omega} \right) = \theta_s(z) - \varepsilon \ T_1 (z). \quad \cdots (33)
\]

where

\[ M_1 = a_{11} \left[ e^{-z \varepsilon} \cos a_z z - e^{-z \varepsilon} \cos a_z z \right] - a_{12} \left[ e^{-z \varepsilon} \sin a_z z - e^{-z \varepsilon} \sin a_z z \right] \]

\[ M_0 = a_{12} \left[ e^{-z \varepsilon} \cos a_z z - e^{-z \varepsilon} \cos a_z z \right] + a_{11} \left[ e^{-z \varepsilon} \sin a_z z - e^{-z \varepsilon} \sin a_z z \right] \]

\[ T_1 = -\frac{e^{-z \varepsilon}}{a_z^2} \left[ a_z \sin a_z z + a_z \cos a_z z \right] \]

\[ c_z = a_z + t a_z, \quad c_z = a_z + t a_z, \quad c_z = a_z + t a_z, \quad c_z = a_z + t a_z, \quad c_z = a_z + t a_z \]

\[ a_1 = \frac{1}{2} \left[ Pr + \sqrt{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2} \right] \left[ \frac{Pr^2 + 4 F Pr}{Pr^2 + 4 F Pr} \right] \left[ \frac{Pr^2 + 4 F Pr}{Pr^2 + 4 F Pr} \right] \]

\[ a_2 = \frac{1}{2} \left[ Pr + \sqrt{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2} \right] \left[ \frac{Pr^2 + 4 F Pr}{Pr^2 + 4 F Pr} \right] \left[ \frac{Pr^2 + 4 F Pr}{Pr^2 + 4 F Pr} \right] \]

\[ a_3 = \frac{1}{2} \left[ 1 + \sqrt{\left(1 + \frac{4}{k} + 4 M^2 \right)^2 + 64 R^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \]

\[ a_4 = \frac{1}{2} \left[ \sqrt{\left(1 + \frac{4}{k} + 4 M^2 \right)^2 + 64 R^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \]

\[ a_z = -\frac{Gr \left( c_z^2 - c_z^2 - c_z^2 \right)}{k} - c_z^2, \quad c_z^2 = c_z^2 - c_z^2 - c_z^2 + 4 R^2 c_z^2 \]

\[ a_k = -\frac{2 Gr R c_z}{c_z^2 - c_z^2 - c_z^2 + 4 R^2 c_z^2} \]

\[ a_7 = \frac{1}{2} \left[ 1 + \sqrt{\left(1 + \frac{4}{k} + 4 M^2 \right)^2 + 16 (2 R + \omega)^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \]

\[ a_8 = \frac{1}{2} \left[ \sqrt{\left(1 + \frac{4}{k} + 4 M^2 \right)^2 + 64 (2 R + \omega)^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \left[ \frac{1 + \frac{4}{k} + 4 M^2}{1 + \frac{4}{k} + 4 M^2} \right] \]

\[ a_9 = a_z^2 - a_z^2 - a_z - \frac{1}{k} - M^2, \]

\[ a_{10} = 2 a_z a_z - 2 R - \omega, \quad a_{11} = \frac{Gr \left( a_z a_z - a_z a_z \right)}{a_z a_z - a_z a_z}, \quad a_{12} = \frac{Gr \left( a_z a_z + a_z a_z \right)}{a_z a_z + a_z a_z}. \]
5. Discussion and Conclusions

From equation (22), it has been found that steady part of the mean primary velocity field has a two layer character. These layers may be identified as suction layer and thermal layer. The suction layer is due to rotation and porosity of the medium. The thermal layer is arising due to interaction of the thermal field due to radiation and the velocity field and is dependent on Prandtl Number and Radiation Parameter.

For physical interpretation of the problem, the numerical values of the mean primary and mean secondary velocity profiles have been computed for fixed values of physical parameter, for Grashof number, Gr = 2, Prandtl number Pr=0.71 (air), frequency of fluctuation \( \omega = 0.5 \) and for different values of Rotation parameter R, Radiation parameter F, the magnetic field parameter M (Hartmann number) and permeability of porous medium \( k \). From fig.1 we observe that the mean primary velocity increases with increase in either rotation parameter R or permeability \( k \). This shows that the viscosity and rotation of porous medium exert retarding influence on the primary flow. It has also been observed that it decreases with increasing radiation parameter F. The mean primary velocity increases in the vicinity of the surface and than decreases with perpendicular distance from the surface. It is interesting to note that mean primary velocity is sufficiently small with increasing intensity of magnetic field parameter.

The mean secondary velocity is given in fig.2 for fixed values of Gr = 2, Pr = 0.71 (air) and \( \omega = 0.5 \). It is observed that it decreases sufficiently higher with increasing radiation parameter F. It is interesting to note that mean secondary velocity increases with rotation parameter and permeability for \( z < 0.5 \), while reverse phenomena is observed for \( z > 0.5 \), showing that the effect of flow parameter is more significant for relatively small values of \( z \). The transient primary velocity profiles are presented in fig.3 for fixed values of Gr = 2, Pr = 0.71 (air) and \( \omega = 0.5 \). It is observed that the transient primary velocity increases with increasing either permeability or rotation parameter. It is interesting to note that it increases with increasing radiation parameter for \( z < 0.9 \) than it decreases for \( z \geq 0.9 \).

The transient secondary velocity is given in fig.4. It is observed that transient velocity increases in the vicinity of vertical surface while, reverse effect is observed with increasing radiation parameter. It is also observed that it increases with increasing rotation parameter and permeability for small value of \( z \), while it decreases with higher value of \( z \). For the nonconducting medium the transient secondary velocity decreasing and attains minima with increasing radiation parameter F.
An increase in $M$ from zero (nonconducting medium) to $M=2$ (conducting fluid) leads to a distinct reduction in the mean and transient velocities (fig.1-4). When $M$ rises, the magnetic body force $\left( -M^2 U \right)$ in the momentum equations (13) is amplified, which retards the flow. Thus, the imposition of an external magnetic field is a powerful mechanism for inhibiting the flow in such a regime. The maximum value of flow velocities take place close to the plate surface at a short distance from it; further in the boundary layer the profiles converge, i.e., the magnetic body force exerts a weaker effect on the flow.

Fig.5 shows the variation of mean and transient temperature profiles for different values of $\omega$ and $F$. The mean temperature and transient temperature both decreases with increasing radiation parameter. Also the transient temperature increases with increasing $\omega$. It is interesting to note that both are decreasing exponentially with distance far away from the vertical surface.

**REFERENCES**


