Robust Stability Analysis for T-S Fuzzy Neural Networks with Time-varying Delays

Pin-Lin Liu

Department of Automation Engineering Institute of Mechatronoptic System, Chienkuo Technology University, Taiwan

*Corresponding author: lp1@cc.ctu.edu.tw

Copyright © 2014 Horizon Research Publishing All rights reserved.

Abstract In this paper, the robust stability of T-S fuzzy uncertain system for neural networks with time-varying delays is investigated. The constraint on the time-varying delay function is removed, which means that a fast time-varying delay is allowed. Based on the Lyapunov-Krasovskii functional techniques and integral inequality approach (IIA), novel robust stability criteria have been derived in terms of linear matrix inequalities which can be easily solved using the efficient convex optimization algorithm. By taking the relationship among the time-varying delay, its upper bound and their difference into account, some less conservative LMI-based delay-dependent stability criteria are obtained without ignoring any useful terms in the derivative of Lyapunov-Krasovskii functional. Examples are included to illustrate our results. These results are shown to be less conservative than those reported in the literature.

Keywords T-S Fuzzy Systems, Time-Varying Delay, Lyapunov-Krasovskii Functional, Linear Matrix Inequalities (Lmis), Integral Inequality Approach (IIA)

1. Introduction

There has been a large amount of literature that studies the stability properties of linear time delay systems. Some fundamental results on these topics have been published in the literature [1-16] and the reference therein. Recently, people have paid more and more attention on the robust stability of T-S fuzzy system with time delay [2] and time-varying delay [1, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16]. Unfortunately, the existing results always assume that the time-varying delay function is continuously differentiable and its derivative is smaller than one, see [7] for example, which is a rigorous constraint. Therefore, it is interesting but challenging to develop the robust stability condition without any constraint on the time-varying delay.

Recently, Fuzzy logic theory has shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear

systems. The well known T-S fuzzy model [14] is a popular and convenient tool in functional approximations. During the last decade, the problem of stability analysis and control synthesis for systems in T-S fuzzy model with time delay have been studied extensively and lot of research works have been reported in the literature [3, 4, 7-11, 15, 16].

More recently, Neural network (NN) have been extensively studied recurrently and involved in many different applications such as signal processing, optimization, fixed point computations, image processing, and other areas [5]. These applications are built upon the stability of the equilibrium of neural networks. Thus, the stability analysis is a necessary step for the design and applications of neural networks. For example, when a neural network is applied to solve the optimization problem, it must have the unique equilibrium which is globally stable. Therefore, stability analysis of neural networks has received much attention and various stability conditions have been obtained. It has been realized that significant time delays as a source of instability and bad performance may occur in neural processing and signal transmission. In contrast to pure neural networks or fuzzy systems, the fuzzy neural networks possess both their advantages. It combines the capability of fuzzy reasoning in handling uncertain information and the capability of artificial neural networks in learning from process. In the last decade, the concept of incorporating fuzzy logic into neural network has grown into a popular research topic [1, 5, 6].

Stability criteria for T-S fuzzy systems are generally classified into two types: delay-dependent and delay-independent. Since delay-dependent criteria make use of information on the lengths of delays, they are less conservative than delay-independent ones, especially when the delay is small. Moreover, the stability of uncertain T-S fuzzy systems with a time-varying delay was studied in [7]. However, there is still room for further investigation. For example, $\dot{x}^T(t)hZ\dot{x}(t) - \int_{t-h(t)}^t \dot{x}^T(s)Z\dot{x}(s)ds$ was used as an estimate of the derivative of $\int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s)Z\dot{x}(s)dsd\theta$, where $0 \leq h(t) \leq h$ and the term $\int_{t-h}^{t-h(t)} \dot{x}^T(s)Z\dot{x}(s)ds$ was ignored, which may lead to considerable conservativeness. The integral inequality approach (IIA) [12, 13] has recently been

devised to study the stability of time-delay systems, and less conservative stability criteria have been derived.

In this paper, we discuss the stability problem for T-S fuzzy uncertain system for neural networks with time-varying delays by employing an integral inequality approach (IIA). Under considering the relationship among the time-varying delay, its upper bound and their difference,

some improved LMI-based stability criteria for uncertain T-S fuzzy uncertain system for neural networks with time-varying delays are obtained without ignoring any useful terms in the derivative of a Lyapunov functional. Finally, numerical examples are given to demonstrate the effectiveness and merits of the proposed method.

2. Stability Description and Preliminaries

The recurrent neural networks with time-varying delay in this paper can be described by the following normalized equations:

$$\dot{u}(t) = -(C + \Delta C(t))u(t) + (A + \Delta A(t))f(u(t)) + (B + \Delta B(t))f(u(t - h(t))) + J, \quad (1)$$

where $u(t) = [u_1(t), \dots, u_n(t)]^T \in R^n$ is the state vector with the n neurons; $f(u(t)) = [f_1(u_1(t)), \dots, f_n(u_n(t))]^T \in R^n$ is called an activation function indicating how the j -th neuron responses to its input; $C = \text{diag}(c_1, \dots, c_n)$ is a diagonal matrix with each $c_i > 0$ controlling the rate with which the i -th unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $A = (a_{ij})_{n \times n}$, and $B = (b_{ij})_{n \times n}$ are the feedback and the delayed feedback matrix, respectively; $J = [J_1, \dots, J_n]^T \in R^n$ is a constant input vector, $\Delta A(t)$, $\Delta B(t)$, and $\Delta C(t)$ are unknown matrices that represent the time-varying parameter uncertainties and $h(t)$ is the time delay of the system satisfies

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq h_d, \quad (2)$$

where h and h_d are some positive constants.

In this paper, the neuron activation functions are assumed to be bounded and satisfy the following assumption.

Assumption 1. It is assumed that each of the activation functions $f_j (j = 1, 2, \dots, n)$ possess the following condition

$$\gamma_i \leq \frac{f_i(\varsigma_1) - f_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq k_i, \quad \varsigma_1 \neq \varsigma_2 \in R, i = 1, 2, \dots, n, \quad (3)$$

where γ_i and k_i are known constant scalars.

Remark 1. If the neuron activation functions satisfy Assumption 1, then they satisfy

$$|f_i(\varsigma_1) - f_i(\varsigma_2)| \leq \max\{|\gamma_i|, |k_i|\} |\varsigma_1 - \varsigma_2| = \rho_i |\varsigma_1 - \varsigma_2|, i = 1, 2, \dots, n. \quad (4)$$

However, we shall point out that this assumption is much *strong* and may lead to conservative conditions for the delay-dependent stability analysis of delayed neural networks. For example, if $\gamma_i < k_i < 0$, then the delay-dependent stability result obtained by using (4) is generally less conservative than the one obtained by using (5). This will be shown via numerical examples in Sec. 4 in this paper.

Next, the equilibrium point $u^* = [u_1^*, \dots, u_n^*]^T$ of system (1) is shifted to the origin through the transformation $x(t) = u(t) - u^*$, then system (1) can be equivalently written as the following system

$$\dot{x}(t) = -(C + \Delta C(t))x(t) + (A + \Delta A(t))g(u(t)) + (B + \Delta B(t))g(u(t - h(t))), \quad (5)$$

where $x(\cdot) = [x_1(\cdot), \dots, x_n(\cdot)]^T$, $g(x(\cdot)) = [g_1(x_1(\cdot)), \dots, g_n(x_n(\cdot))]^T$, $g_i(x_i(\cdot)) = f_i(x_i(\cdot) + u_i^*) - f_i(u_i^*)$, $i = 1, 2, \dots, n$.

The matrices $\Delta C_i(t)$, $\Delta A_i(t)$ and $\Delta B_i(t)$ $i = 1, 2, \dots, r$ are the uncertainties of the system and have the form

$$[\Delta C_i(t) \quad \Delta A_i(t) \quad \Delta B_i(t)] = D_i F(t) [E_{ci} \quad E_{ai} \quad E_{bi}], \quad (6)$$

where D_i , E_{ci} , E_{ai} , and E_{bi} are known constant real matrices with appropriate dimensions and $F(t)$ is an unknown matrix function with Lebesgue-measurable elements bounded by

$$F^T(t)F(t) \leq I, \quad \forall t, \quad (7)$$

where I is an appropriately dimensioned identity matrix.

The fuzzy model of (5) described by

Plant Rule i : If $z_1(t)$ is M_{i1} , and ..., and $z_p(t)$ is M_{ip} then

$$\dot{x}(t) = -(C_i + \Delta C_i(t))x(t) + (A_i + \Delta A_i(t))g(x(t)) + (B_i + \Delta B_i(t))g(x(t-h(t))), \quad (8a)$$

$$x(t) = \phi(t), t \in [-h, 0], i = 1, 2, \dots, r, \quad (8b)$$

where $z_1(t), z_2(t), \dots, z_p(t)$ are the premise variables; $M_{ij}, i = 1, 2, \dots, r, j = 1, 2, \dots, p$ are the fuzzy sets; $x(t) \in R^n$ is the state vector; $\phi(t)$ is a vector-valued initial condition; the scalars r is the number of IF-Then rules.

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^r w_i(z(t))[-(C_i + \Delta C_i(t))x(t) + (A_i + \Delta A_i(t))g(x(t)) + (B_i + \Delta B_i(t))g(x(t-h(t)))]}{\sum_{i=1}^r w_i(z(t))} \\ &= \sum_{i=1}^r u_i(z(t))[-(C_i + \Delta C_i(t))x(t) + (A_i + \Delta A_i(t))g(x(t)) + (B_i + \Delta B_i(t))g(x(t-h(t)))] \\ &= -\bar{C}_i x(t) + \bar{A}_i g(x(t)) + \bar{B}_i g(x(t-h(t))), \end{aligned} \quad (9a)$$

$$x(t) = \phi(t), t \in [-h, 0], \quad (9b)$$

where $z = [z_1, z_2, \dots, z_p]$; $w_i : R^p \rightarrow [0, 1], i = 1, 2, \dots, r$, is the membership function of the system with respect to the plant rule

$$i; u_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^r w_i(z(t))}; \bar{C}_i = \sum_{i=1}^r u_i(z(t))(C_i + \Delta C_i(t)) \quad \bar{A}_i = \sum_{i=1}^r u_i(z(t))(A_i + \Delta A_i(t)), \quad \bar{B}_i = \sum_{i=1}^r u_i(z(t))(B_i + \Delta B_i(t)).$$

It is assumed that $w_i(z(t)) \geq 0, i = 1, 2, \dots, r, \sum_{i=1}^r w_i(z(t)) \geq 0, \forall t$, so we have $u_i(z(t)) \geq 0, \sum_{i=1}^r u_i(z(t)) = 1$.

In the following, we will develop some practically computable stability criteria for the system described(9).The following lemmas are useful in deriving the criteria.

First, we introduce the following integral inequality approach (IIA), which be used in the proof of ours.

Lemma 1 [12, 13]. For any positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0, \quad (10a)$$

the following integral inequality holds

$$\begin{aligned} & -\int_{t-h(t)}^t \dot{x}^T(s) X_{33} \dot{x}(s) ds \\ & \leq \int_{t-h(t)}^t \begin{bmatrix} x^T(t) & x^T(t-h(t)) & \dot{x}^T(s) \end{bmatrix} \times \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds. \end{aligned} \quad (10b)$$

Secondary, the following Schur complement result, which is essential in the proofs of Theorem 1, is introduced.

Lemma 2 [2]. The following matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0, \quad (11a)$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$ and $S(x)$ depend affine on x , is equivalent to

$$R(x) < 0, \quad (11b)$$

$$Q(x) < 0, \quad (11c)$$

and

$$Q(x) - S(x)R^{-1}(x)S^T(x) < 0. \quad (11d)$$

Finally, the following Lemma 3 will be used to handle the parametrical perturbation.

Lemma 3 [2]. Given symmetric matrices Ω and D, E , of appropriate dimensions,

$$\Omega + DF(t)E + E^T F^T(t)D^T < 0, \quad (12a)$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exists some $\varepsilon > 0$ such that

$$\Omega + \varepsilon D D^T + \varepsilon^{-1} E^T E < 0. \quad (12b)$$

3. Main Results

In this section, we use the integral inequality approach (IIA) to obtain stability criterion for a T-S fuzzy neural networks with time-varying delays. First, we take up the case where $\Delta A_i(t) = 0$ and $\Delta B_i(t) = 0$ in system (9) as follows:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-h(t))), \quad (13a)$$

$$x(t) = \phi(t), t \in [-h, 0], \quad (13b)$$

where $C = \sum_{i=1}^r u_i(z(t))C_i$, $A = \sum_{i=1}^r u_i(z(t))A_i$, $B = \sum_{i=1}^r u_i(z(t))B_i$.

Based on the Lyapunov-Krasovskii stability theorem and integral inequality approach (IIA), the following result is obtained.

Theorem 1. For given positive scalars h and h_d , the nominal T-S fuzzy neural networks with time-varying delays (13) is asymptotically stable if there exist symmetry positive-definite matrices $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T > 0$, diagonal Y matrices $S \geq 0$, $U \geq 0$, $V \geq 0$, positive semi-definite matrices $X = \begin{bmatrix} X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$, and $Y = \begin{bmatrix} Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} \geq 0$, such that the following LMIs hold for $i = 1, 2, \dots, r$,

$$\Omega_i = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & \Omega_{16} \\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} & 0 & 0 & \Omega_{26} \\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} & \Omega_{34} & 0 & \Omega_{36} \\ \Omega_{14}^T & 0 & \Omega_{34}^T & \Omega_{44} & \Omega_{45} & 0 \\ 0 & 0 & 0 & \Omega_{45}^T & \Omega_{55} & 0 \\ \Omega_{16}^T & \Omega_{26}^T & \Omega_{36}^T & 0 & 0 & \Omega_{66} \end{bmatrix} < 0 \quad (14a)$$

and

$$Z - X_{33} \geq 0 \quad (14b)$$

$$Z - Y_{33} \geq 0 \quad (14c)$$

where

$$\begin{aligned} K &= \text{diag}\{k_1, k_2, \dots, k_n\}, \Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}, \\ \Omega_{11} &= -C_i^T P - P C_i - \Gamma U K + \Gamma S C_i + C_i^T S \Gamma + Q + R + h X_{11} + X_{13} + X_{13}^T, \\ \Omega_{12} &= P A_i - C_i^T S - \Gamma S A_i + \frac{U(\Gamma + K)}{2}, \\ \Omega_{13} &= P B_i - \Gamma S B_i, \Omega_{14} = h X_{12} - X_{13} + X_{23}^T, \\ \Omega_{16} &= -h C_i^T Z, \Omega_{22} = -U + A_i^T S + S A_i, \Omega_{23} = S B_i, \\ \Omega_{26} &= h A_i^T Z, \Omega_{33} = -V, \\ \Omega_{34} &= \frac{V(\Gamma + K)}{2}, \Omega_{36} = h B_i^T Z, \\ \Omega_{44} &= -(1 - h_d)Q - \Gamma V K + h X_{22} - X_{23} - X_{23}^T + h Y_{11} + Y_{13} + Y_{13}^T, \\ \Omega_{45} &= h Y_{12} - Y_{13} + Y_{23}^T, \Omega_{55} = -R + h Y_{22} - Y_{23} - Y_{23}^T, \Omega_{66} = -h Z. \end{aligned}$$

Proof: Choose the following fuzzy Lyapunov-Krasovskii functional candidate to be

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \quad (15)$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t) \\
V_2(t) &= 2 \sum_{i=1}^n s_i \int_0^{x_i(t)} (g_i(s) - r_i s) ds, \\
V_3(t) &= \int_{t-h(t)}^t x^T(s)Qx(s)ds + \int_{t-h}^t x^T(s)Rx(s)ds \\
V_4(t) &= \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s)Z\dot{x}(s)dsd\theta
\end{aligned}$$

Then, taking the time derivative of $V(t)$ with respect to t along the system (13) yield

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \quad (16)$$

where

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
&= x^T(t)(-C^TP - PC)x(t) + 2x^T(t)P[Ag(x(t)) + Bg(x(t-h(t)))],
\end{aligned} \quad (17)$$

$$\begin{aligned}
\dot{V}_2(t) &= 2[g^T(x(t)) - x^T(t)\Gamma]S\dot{x}(t) \\
&= 2[g^T(x(t)) - x^T(t)\Gamma]S[-Cx(t) + Ag(x(t)) + Bg(x(t-h(t)))]
\end{aligned} \quad (18)$$

$$\begin{aligned}
\dot{V}_3(t) &= x^T(t)Qx(t) - (1 - \dot{h}(t))x^T(t-h(t))Qx(t-h(t)) \\
&\quad + x^T(t)Rx(t) - x^T(t-h)Rx(t-h) \\
&\leq x^T(t)Qx(t) - (1 - h_d)x^T(t-h(t))Qx(t-h(t)) \\
&\quad + x^T(t)Rx(t) - x^T(t-h)Rx(t-h),
\end{aligned} \quad (19)$$

and

$$\dot{V}_4(t) = \dot{x}^T(t)hZ\dot{x}(t) - \int_{t-h}^t \dot{x}^T(s)Z\dot{x}(s)ds. \quad (20)$$

Alternatively, the following equations are true:

$$\begin{aligned}
& -\int_{t-h}^t \dot{x}^T(s)Z\dot{x}(s)ds \\
&= -\int_{t-h(t)}^t \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-h}^{t-h(t)} \dot{x}^T(s)Z\dot{x}(s)ds \\
&= -\int_{t-h(t)}^t \dot{x}^T(s)(Z - X_{33})\dot{x}(s)ds - \int_{t-h(t)}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds \\
&\quad - \int_{t-h}^{t-h(t)} \dot{x}^T(s)(Z - Y_{33})\dot{x}(s)ds - \int_{t-h}^{t-h(t)} \dot{x}^T(s)Y_{33}\dot{x}(s)ds.
\end{aligned} \quad (21)$$

Using Lemma 1, the term $-\int_{t-h(t)}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds$ can be written that

$$\begin{aligned}
& -\int_{t-h(t)}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds \\
&\leq \int_{t-h(t)}^t \begin{bmatrix} x^T(t) & x^T(t-h(t)) & \dot{x}^T(s) \end{bmatrix} \times \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds \\
&\leq x^T(t)[hX_{11} + X_{13}^T + X_{13}]x(t) + x^T(t)[hX_{12} - X_{13} + X_{23}^T]x(t-h(t)) \\
&\quad + x^T(t-h(t))[hX_{12}^T - X_{13}^T + X_{23}]x(t) + x^T(t-h(t))[hX_{22} - X_{23} - X_{23}^T]x(t-h(t)).
\end{aligned} \quad (22)$$

Similarly, we have

$$\begin{aligned}
& -\int_{t-h}^{t-h(t)} \dot{x}^T(s)Y_{33}\dot{x}(s)ds \\
&\leq x^T(t-h(t))[hY_{11} + Y_{13}^T + Y_{13}]x(t-h(t)) + x^T(t-h(t))[hY_{12} - Y_{13} + Y_{23}^T]x(t-h) \\
&\quad + x^T(t-h)[hY_{12}^T - Y_{13}^T + Y_{23}]x(t-h(t)) + x^T(t-h)[hY_{22} - Y_{23} - Y_{23}^T]x(t-h).
\end{aligned} \quad (23)$$

Evaluating $\dot{x}^T(t)hZ\dot{x}(t)$ along solution to (13), gives as follows:

$$\begin{aligned}
& \dot{x}^T(t)hZ\dot{x}(t) = \\
& [-Cx(t) + Ag(x(t)) + Bg(x(t-h(t)))]^T (hZ) \times [-Cx(t) + Ag(x(t)) + Bg(x(t-h(t)))] \\
&= x^T(t)hC^TZCx(t) - x^T(t)hC^TZA g(x(t)) - x^T(t)hC^TZB g(x(t-h(t))) \\
&\quad - g^T(x(t))hA^TZCx(t) + g^T(x(t))hA^TZA g(x(t)) + g^T(x(t))hA^TZB g(x(t-h(t))) \\
&\quad - g^T(x(t-h(t)))hA^TZCx(t) + g^T(x(t-h(t)))hA^TZA g(x(t)) + g^T(x(t-h(t)))hA^TZB g(x(t-h(t))).
\end{aligned} \quad (24)$$

By (4), it can be verified that

$$\begin{aligned} 0 &= g^T(x(t))Ug(x(t)) - g^T(x(t))Ug(x(t)) \\ &= -x^T(t)\Gamma UKx(t) + x^T(t)U(\Gamma + K)g(x(t)) - g^T(x(t))Ug(x(t)) \end{aligned} \quad (25)$$

Similarly, there holds

$$\begin{aligned} 0 &= g^T(x(t-h(t)))Vg(x(t-h(t))) - g^T(x(t-h(t)))Vg(x(t-h(t))) \\ &= -x^T(t-h(t))\Gamma VKx(t-h(t)) + x^T(t-h(t))V(\Gamma + K)g(x(t-h(t))) \\ &\quad - g^T(x(t-h(t)))Vg(x(t-h(t))) \end{aligned} \quad (26)$$

Substituting the above equations (17)-(26) into (16), we obtain

$$\dot{V}(t) \leq \xi^T(t)\Xi\xi(t) - \int_{t-h(t)}^t \dot{x}^T(s)(Z - X_{33})\dot{x}(s)ds - \int_{t-h}^{t-h(t)} \dot{x}^T(s)(Z - Y_{33})\dot{x}(s)ds, \quad (27)$$

where $\xi^T(t) = [x^T(t) \quad g^T(x(t)) \quad g^T(x(t-h(t))) \quad x^T(t-h(t)) \quad x^T(t-h)]$ and $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 \\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} & 0 & 0 \\ \Xi_{13}^T & \Xi_{23}^T & \Xi_{33} & \Xi_{34} & 0 \\ \Xi_{14}^T & 0 & \Xi_{34}^T & \Xi_{44} & \Xi_{45} \\ 0 & 0 & 0 & \Xi_{45}^T & \Xi_{55} \end{bmatrix} < 0$, with

$$\begin{aligned} K &= \text{diag}\{k_1, k_2, \dots, k_n\}, \Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}, \\ \Xi_{11} &= -C^T P - PC - \Gamma UK + \Gamma SC + C^T S \Gamma + Q + R + hX_{11} + X_{13} + X_{13}^T + hC^T ZC, \\ \Xi_{12} &= PA - C^T S - \Gamma SA + \frac{U(\Gamma + K)}{2} - hC^T ZA, \\ \Xi_{13} &= PB - \Gamma SB - hC^T ZB, \\ \Xi_{14} &= hX_{12} - X_{13} + X_{23}^T, \Xi_{22} = -U + A^T S + SA + hA^T ZA, \\ \Xi_{23} &= SB + hA^T ZB, \Xi_{33} = -V + hB^T ZB, \\ \Xi_{34} &= \frac{V(\Gamma + K)}{2}, \\ \Xi_{44} &= -(1-h_d)Q - \Gamma VK + hX_{22} - X_{23} - X_{23}^T \\ &\quad + hY_{11} + Y_{13} + Y_{13}^T, \\ \Xi_{45} &= hY_{12} - Y_{13} + Y_{23}^T, \Xi_{55} = -R + hY_{22} - Y_{23} - Y_{23}^T. \end{aligned}$$

Finally, using the Schur complements of Lemma 2, with some effort we can show that (27) guarantees of $\dot{V}(t) < -\delta \|x(t)\|^2$ for a sufficiently small $\delta > 0$. It is clear that if $\Xi < 0$, $Z - X_{33} \geq 0$, and $Z - Y_{33} \geq 0$. Furthermore, (14) implies $\sum_{i=1}^r u_i(z(t))\Omega_i < 0$, which is equivalent to (27). Therefore, if LMIs (14) are feasible, the system (13) is asymptotically stable. This completes the proof. $\square$

Based on Theorem 1, we have the following result for uncertain T-S fuzzy system (9).

Theorem 2. For given positive scalars h and h_d , the uncertain T-S fuzzy neural network with time-varying delay (9) is asymptotically stable if there exist symmetric positive-definite matrices $P = P^T > 0, Q = Q^T > 0, R = R^T > 0, Z = Z^T > 0$, diagonal matrices $S \geq 0, U \geq 0, V \geq 0$, a scalar $\varepsilon_i > 0$ and positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0, Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12}^T & Y_{22} & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_{33} \end{bmatrix} \geq 0,$$

such that the following LMIs are true for $i = 1, 2, \dots, r$:

$$\bar{\Omega}_i = \begin{bmatrix} \Omega_{11} + \varepsilon_i E_{ci}^T E_{ci} & \Omega_{12} - \varepsilon_i E_{ci}^T E_{ai} & \Omega_{13} - \varepsilon_i E_{ci}^T E_{bi} & \Omega_{14} & 0 & \Omega_{16} & PD_i \\ \Omega_{12}^T - \varepsilon_i E_{ai}^T E_{ai} & \Omega_{22} + \varepsilon_i E_{ai}^T E_{ai} & \Omega_{23} + \varepsilon_i E_{ai}^T E_{bi} & 0 & 0 & \Omega_{26} & 0 \\ \Omega_{13}^T - \varepsilon_i E_{bi}^T E_{ai} & \Omega_{23}^T + \varepsilon_i E_{bi}^T E_{ai} & \Omega_{33} + \varepsilon_i E_{bi}^T E_{bi} & \Omega_{34} & 0 & \Omega_{36} & 0 \\ \Omega_{14}^T & 0 & \Omega_{34}^T & \Omega_{44} & \Omega_{45} & 0 & 0 \\ 0 & 0 & 0 & \Omega_{45}^T & \Omega_{55} & 0 & 0 \\ \Omega_{16}^T & \Omega_{26}^T & \Omega_{36}^T & 0 & 0 & \Omega_{66} & hZD_i \\ D_i^T P & 0 & 0 & 0 & 0 & hD_i^T Z & -\varepsilon_i I \end{bmatrix} < 0, \quad (28a)$$

and

$$Z - X_{33} \geq 0, \quad (28b)$$

$$Z - Y_{33} \geq 0, \quad (28c)$$

where $\Omega_{ij}, (i, j = 1, \dots, 6; i < j \leq 6)$ are defined in (14).

It is, incidentally, worth noting that the time delay uncertain system (9) is asymptotically stable, that is, the uncertain parts of the nominal system can be tolerated within allowable time delay h .

Proof: Replacing A_i, B_i , and C_i in (14) with $A_i + D_i F(t) E_{ai}, B_i + D_i F(t) E_{bi}$, and $C_i + D_i F(t) E_{ci}$, respectively, we apply Lemma 2 for system (9) is equivalent to the following condition:

$$\Omega_i + \Gamma_{di}^T F(t) \Gamma_{ei} + \Gamma_{ei}^T F(t) \Gamma_{di} < 0, \quad (29)$$

where $\Gamma_{di} = [PD_i \ 0 \ 0 \ 0 \ 0 \ hZD_i]^T$ and $\Gamma_{ei} = [E_{ci} \ E_{ai} \ E_{bi} \ 0 \ 0 \ 0]$.

According to Lemma 3, (29) is true if there exist a scalar $\varepsilon_i > 0$ such that the following inequality holds

$$\Omega_i + \varepsilon_i^{-1} \Gamma_{di}^T \Gamma_{di} + \varepsilon_i \Gamma_{ei}^T \Gamma_{ei} < 0. \quad (30)$$

Applying the Schur complement shows that (30) is equivalent to (28a). This completes the proof.

If the upper bound of the derivative of time-varying delay h_d is unknown, Theorem 2 can be reduced to the result with $Q = 0$ and $Y = 0$, we have the following Corollary 1.

Corollary 1. Consider system (9) with constant delay. For given a positive scalar h , the system is asymptotically stable if there exist symmetric positive-definite matrices $P = P^T > 0$, $X = X^T > 0$, $Z = Z^T > 0$, diagonal matrices $S \geq 0, U \geq 0, V \geq 0$, a scalar $\varepsilon_i > 0$ and positive semi-definite matrices $X = \begin{bmatrix} X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$, such that the following LMIs are true for $i = 1, 2, \dots, r$:

$$\bar{\Psi} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & -hC_i^T Z & PD_i \\ \Psi_{12}^T & \Psi_{22} & \Psi_{23} & 0 & hA_i^T Z & 0 \\ \Psi_{13}^T & \Psi_{23}^T & \Psi_{33} & \Psi_{34} & hB_i^T Z & 0 \\ \Psi_{14}^T & 0 & \Psi_{34}^T & \Psi_{44} & 0 & 0 \\ -hZC_i & hZA_i & hZB_i & 0 & -hZ & hZD_i \\ D_i^T P & 0 & 0 & 0 & hD_i^T Z & -\varepsilon_i I \end{bmatrix} < 0, \quad (31a)$$

and

$$Z - X_{33} \geq 0, \quad (31b)$$

where

$$\begin{aligned} K &= \text{diag}\{k_1, k_2, \dots, k_n\}, \Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}, \\ \Psi_{11} &= -C_i^T P - PC_i - \Gamma UK + \Gamma SC_i + C_i^T S \Gamma + R + hX_{11} + X_{13} + X_{13}^T, \\ \Psi_{12} &= PA_i - C_i^T S - \Gamma SA_i + \frac{U(\Gamma + K)}{2}, \Psi_{13} = PB_i - \Gamma SB_i, \\ \Psi_{14} &= hX_{12} - X_{13} + X_{23}^T, \\ \Psi_{22} &= -U + A_i^T S + SA_i, \Psi_{23} = SB_i, \Psi_{33} = -V, \\ \Psi_{34} &= \frac{V(\Gamma + K)}{2}, \\ \Psi_{44} &= -\Gamma VK + hX_{22} - X_{23} - X_{23}^T + hY_{11} + Y_{13} + Y_{13}^T. \end{aligned}$$

Proof. If the matrix $Q = 0$ is selected in (15). This proof can be completed in a similar formulation to Theorem 1 and

Theorem 2.

Remark 1: Theorem 2 provides delay-dependent robust asymptotic stability criterion for the time delay uncertain systems (9) in terms of solvability of LMIs [2]. Based on them, we can obtain the maximum allowable delay bound (MADB) $\bar{h}$ such that (9) is stable by solving the following convex optimization problem

$$\begin{cases} \text{Maximize} & \bar{h} \\ \text{Subject to} & (28) \end{cases} \quad (32)$$

Inequality (32) is a convex optimization problem and can be obtained efficiently using the MATLAB LMI Toolbox.

4. Illustrative Examples

In this section, the following examples are provided to show the effectiveness of the proposed methods.

Example 1. Consider a time delayed fuzzy system without uncertainty. The T-S fuzzy model of this fuzzy neural network with time-varying delay is of the following form:

Plant rules:

Rule 1: If $x_1(t)$ is M_1 , then

$$\dot{x}(t) = -C_1 x(t) + A_1 g(x(t)) + B_1 g(x(t-h(t))), \quad (33a)$$

Rule 2: If $x_1(t)$ is M_2 , then

$$\dot{x}(t) = -C_2 x(t) + A_2 g(x(t)) + B_2 g(x(t-h(t))), \quad (33b)$$

and the membership function for rule 1 and rule 2 are

$$M_1(x_1(t)) = \frac{1}{1 + \exp(-2x_1(t))}, M_2(x_1(t)) = 1 - M_1(x_1(t)),$$

where

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 0.3 & -0.4 \\ 0.28 & 0.7 \end{bmatrix}, B_1 = \begin{bmatrix} -0.2 & 0.2 \\ -0.3 & 0.12 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.5 & -0.26 \\ 0.28 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & -0.02 \\ -0.22 & 0.1 \end{bmatrix}.$$

The neuron activation functions are assumed to satisfy Assumption 1 with $\Gamma = \text{diag}\{\gamma_1, \gamma_2\}$ and $K = \text{diag}\{1, 1\}$.

Solution: For $\gamma_1 = \gamma_2 = 0$, and $h_d = 1.5$, the constraint $\dot{h}(t) \leq h_d < 1$ is essential in [6], but it is not necessary in our result.

The MADB $\bar{h}$ that guarantees the system (33) to be asymptotically stable is calculated to be $\bar{h} = 1.1503$ in [5], which is $\bar{h} = 2.6853$ by using Theorem 1 in this paper. It is seen that our results improve the existing results [5, 6]. In case of $\bar{h} = 2.6853$, solving Theorem 1 yields the following set of feasible solutions

$$\begin{aligned} P &= \begin{bmatrix} 3.8082 & 0.4177 \\ 0.4177 & 2.2776 \end{bmatrix}, Q = \begin{bmatrix} 0.2558 & 0.0286 \\ 0.0286 & 0.1353 \end{bmatrix}, R = \begin{bmatrix} 2.1114 & 0.3075 \\ 0.3075 & 0.8203 \end{bmatrix}, Z = \begin{bmatrix} 0.9382 & 0.0604 \\ 0.0604 & 0.8101 \end{bmatrix}, \\ X_{11} &= \begin{bmatrix} 0.5792 & 0.0940 \\ 0.0940 & 0.2013 \end{bmatrix}, X_{12} = \begin{bmatrix} -0.1079 & -0.0054 \\ -0.0056 & -0.1005 \end{bmatrix}, X_{13} = \begin{bmatrix} -0.1449 & 0.0007 \\ -0.0007 & -0.1671 \end{bmatrix}, X_{22} = \begin{bmatrix} 0.0922 & 0.0055 \\ 0.0055 & 0.0871 \end{bmatrix}, \\ X_{23} &= \begin{bmatrix} 0.2011 & 0.0120 \\ 0.0120 & 0.1783 \end{bmatrix}, X_{33} = \begin{bmatrix} 0.5827 & 0.0366 \\ 0.0366 & 0.5132 \end{bmatrix}, Y_{11} = \begin{bmatrix} 0.0956 & 0.0073 \\ 0.0073 & 0.0833 \end{bmatrix}, Y_{12} = \begin{bmatrix} -0.0629 & -0.0020 \\ -0.0017 & -0.0649 \end{bmatrix}, \\ Y_{13} &= \begin{bmatrix} -0.2204 & -0.0142 \\ -0.0141 & -0.1934 \end{bmatrix}, Y_{22} = \begin{bmatrix} 0.4898 & 0.0703 \\ 0.0703 & 0.2097 \end{bmatrix}, Y_{23} = \begin{bmatrix} 0.1728 & 0.0046 \\ 0.0056 & 0.1793 \end{bmatrix}, Y_{33} = \begin{bmatrix} 0.6155 & 0.0398 \\ 0.0398 & 0.5413 \end{bmatrix}, \\ S &= \begin{bmatrix} 1.3511 & 0 \\ 0 & 1.2298 \end{bmatrix}, U = \begin{bmatrix} 2.6277 & 0 \\ 0 & 4.1331 \end{bmatrix}, V = \begin{bmatrix} 2.7562 & 0 \\ 0 & 2.0284 \end{bmatrix}. \end{aligned}$$

Therefore, the fuzzy neural networks with time-varying delays (33) are globally asymptotically stable.

For $\gamma_1 = 0.2, \gamma_2 = -0.1$, applying Theorem 1, the MADB $\bar{h}$ that guarantees the system (33) to be asymptotically stable is calculated to be $\bar{h} = 2.3548$.

Example 2. Consider a time delayed fuzzy system without uncertainty. The T-S fuzzy model of this fuzzy system is of the following form:

Plant rules:

Rule 1: If $x_1(t)$ is M_1 , then

$$\dot{x}(t) = -C_1 x(t) + A_1 g(x(t)) + B_1 g(x(t-h(t))), \quad (34a)$$

Rule 2: If $x_1(t)$ is M_2 , then

$$\dot{x}(t) = -C_2 x(t) + A_2 g(x(t)) + B_2 g(x(t-h(t))), \quad (34b)$$

and the membership function for rule 1 and rule 2 are

$$M_1(x_1(t)) = \frac{1}{1 + \exp(-2x_1(t))}, M_2(x_1(t)) = 1 - M_1(x_1(t)),$$

where

$$C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1.8 & 1 \\ 2 & 1.8 \end{bmatrix}, C_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 2.6 & 0 \\ 0 & 2.5 \end{bmatrix}.$$

The neuron activation functions are assumed to satisfy Assumption 1 with $\Gamma = \text{diag}\{\gamma_1, \gamma_2\}$ and $K = \text{diag}\{0.2, 0.2\}$.

Solution: For $\gamma_1 = \gamma_2 = 0$, and $h_d = 0.4$, the MADB $\bar{h}$ that guarantees the system (34) to be asymptotically stable is calculated to be $\bar{h} = 1.8450$ in [1], which is $\bar{h} = 6.13563$ by using Theorem 1 in this paper. It is seen that our results improve the existing results [1].

For $\gamma_1 = 0.1, \gamma_2 = 0.1$, applying Theorem 1, the MADB $\bar{h}$ that guarantees the system (33) to be asymptotically stable is calculated to be $\bar{h} = 5.0178$.

Example 3. Consider a time delayed fuzzy system with uncertainty. The T-S fuzzy model of this fuzzy system is of the following form:

Plant rules:

Rule 1: If $x_1(t)$ is M_1 , then

$$\dot{x}(t) = -(C_1 + D_1 F(t) E_{c1}) x(t) + (B_1 + D_1 F(t) E_{b1}) x(t-h(t)) \quad (35a)$$

Rule 2: If $x_1(t)$ is M_2 , then

$$\dot{x}(t) = -(C_2 + D_2 F(t) E_{c2}) x(t) + (B_2 + D_2 F(t) E_{b2}) x(t-h(t)) \quad (35b)$$

and the membership function for rule 1 and rule 2 are

$$M_1(x_1(t)) = (1 - \frac{1}{1 + \exp(-3(x_2/0.5 - \pi/2))}) \times (\frac{1}{1 + \exp(-3(x_2/0.5 - \pi/2))}),$$

$$M_2(x_1(t)) = 1 - M_1(x_1(t)),$$

where

$$C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 & -0.3 \\ 0.2 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & -0.3 \end{bmatrix}, E_{c1} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, E_{b1} = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix},$$

$$E_{c2} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, E_{b2} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.4 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}.$$

The neuron activation functions are assumed to satisfy Assumption 1 with $\Gamma = \text{diag}\{0, 0\}$ and $K = \text{diag}\{1, 1\}$.

Solution: By taking the parameter $h_d = 0$, we get the Theorem 2 remains feasible for any delay time $h \leq 6.9648$. However, applying criterion in Balasubramaniam and Chandran [1], the maximum value of $\bar{h}$ for the above system is $\bar{h} = 0.4136$. Hence, it is obvious that the results obtained from our simple method are less conservative than those obtained from the existing methods.

Furthermore, by taking the various h_d , and from Theorem 2, we obtain the upper bound of delay time $\bar{h}$ as shown in the Table 1. From the above results of Table 1, if the h_d increases the delay time length decreases. It is shown that when h_d is known, our obtained results are better than those in [8, 9, 11]; when h_d is unknown, the results in [7] fail to verify that the system is stable, while Theorem 2 of this paper can obtain the better upper bounds than those in [7, 8, 9, 11].

Table 1. Maximum allowable delay bound (MADB) with different h_d for example 3

h_d	0	0.01	0.1	0.5	unknown
Li et al. [7]	0.950	0.944	0.892	0.637	-
Lien [8]	1.158	1.155	1.113	0.929	0.443
Lien et al. [9]	1.168	1.163	1.122	0.934	0.4999
Liu et al. [11]	1.353	1.348	1.303	1.147	1.081
Theorem 2	1.5194	1.5066	1.3943	1.1841	1.1424

5. Conclusion

In this paper, some less conservative LMI-based robust stability criteria are obtained without ignoring any terms in the derivative of Lyapunov-Krasovskii functional for uncertain T-S fuzzy neural networks with time-varying delays. Based on the Lyapunov-Krasovskii functional techniques, novel robust stability criteria have been derived in terms of linear matrix inequalities which can be easily solved using the efficient convex optimization algorithm. The LMI optimization approaches are used to obtain sufficient conditions that are very easy to be checked by using the LMI Toolbox in Matlab. Numerical examples demonstrate that the proposed method is an improvement over the existing ones.

REFERENCES

- [1] P. Balasubramaniam and R. Chandran, Delay decomposition approach to stability analysis for uncertain fuzzy Hopfield neural networks with time-varying delay, *Communications in Nonlinear Science and Numerical Simulation*, Vol. 16, pp. 2098-2108, 2011.
- [2] S. L., Boyd, E. L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Studies in Applied Mathematics, SIAM, Philadelphia, USA, 1994.
- [3] C. W. Chen, C. L. Lin, C. H. Tsai, C. Y. Chen, and K. Yeh, A novel delay- dependent criterion for time-delay T-S fuzzy systems using fuzzy Lyapunov method, *International Journal of Artificial Intelligence Tools*, Vol.16, No. 3, pp.545- 552, 2007.
- [4] H. Han, Adaptive fuzzy control for a class of uncertain nonlinear systems via LMI approach, *International Journal of Innovative Computing, Information and Control*, Vol.6, No.1, pp. 275-285, 2010.
- [5] Y. Y. Hou, T. L. Liao, and J. J. Yan, Stability analysis of Takagi–Sugeno fuzzy cellular neural networks with time-varying delays, *IEEE Transactions on Systems, Man, and Cybernetics–Part B: Cybernetics*, Vol. 37, No. 3, pp. 720-726, 2007.
- [6] H. Huang, D. W. C. Ho, and J. Lam, Stochastic stability analysis of fuzzy Hopfield neural networks with time-varying delays, *IEEE Trans. Circuits Syst. I, Reg. Papers*, Vol. 52, No. 5, pp. 251-255, 2005.
- [7] C. G., Li, H. J. Wang, and X. F. Liao, Delay-dependent robust stability of uncertain fuzzy systems with time-varying delays, *IEE Proceedings Control Theory & Applications*, Vol. 151, pp.417-421, 2004.
- [8] C. H. Lien, Further results on delay-dependent robust stability of uncertain fuzzy systems with time-varying delay, *Chaos, Solitons and Fractals*, Vol.28, pp.422-427, 2006.
- [9] C. H. Lien, K.W. Yu, W. D. Chen, Z. L. Wan, and Y. J. Chung, Stability criteria for uncertain Takagi–Sugeno fuzzy systems with interval time-varying delay, *IET Control Theory and Applications*, Vol.1, No. 3, pp.746-769, 2007.
- [10] C. Lin, Q. G. Wang, T. H. Lee, and Y. He, *LMI Approach to analysis and control of Takagi-Sugeno fuzzy systems with time delay*, New York: Springer-Verlag, 2007.
- [11] F. Liu, M. Wu, Y. He, and R. Yokoyama, New delay-dependent stability criteria for T-S fuzzy systems with time-varying delay, *Fuzzy Sets and Systems*, Vol.161, No. 15, pp.2033-2042, 2010.
- [12] P. L. Liu, Global asymptotic stability for neural networks systems with discrete and distributed delay, *ICIC Express Letters, Part B Applications*, vol.1, no.2, pp.233- 240, 2010.
- [13] P. L. Liu, Delay-range-dependent robust stability for uncertain singular systems with interval time-varying delays, *Universal Journal of Control and Automation*, Vol. 1, No.1, pp. 1-9, 2013.
- [14] T. Takagi, and M. Sugeno, Fuzzy identification of systems and its application to modeling and control, *IEEE Transactions Systems Man Cybernetics*, vol.15, pp.116–132, 1985.
- [15] S. H. Tong, W. Zhang, and T. Wang, Robust stabilization conditions and observer-based controllers for fuzzy systems with input delay, *International Journal of Innovative Computing, Information and Control*, Vol.6, No. 12, pp. 5473-5484, 2010.
- [16] Y. Wang, C., Gong, B. Su, and Y. Wang, Delay-dependent robust stability of uncertain T-S fuzzy systems with time-varying delay, *International Journal of Innovative Computing, Information and Control*, Vol.5, No. 9, pp. 2665-2674, 2009.