Difference Cordial Labeling of Subdivision of Snake Graphs

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Abstract Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, \dots, p\}$ be a function. For each edge uv , assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordial labeling behavior of subdivision of some snake graphs.

Keywords Triangular snake, Quadrilateral snake, Subdivision, Double triangular snake, Double quadrilateral snake

1 Introduction

Let $G = (V, E)$ be a (p, q) graph. In this paper we have considered only simple, non-trivial and undirected graphs. The number of vertices of G is called the order of G and the number of edges of G is called the size G . The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uvw . Graph labeling plays a vital role of numerous fields of sciences and some of them are astronomy, coding theory, x-ray crystallography, radar, database management etc.[1]. Ponraj et al. introduced difference cordial labeling of graphs [3]. Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, \dots, p\}$ be a map. For each edge uv , assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with admits difference cordial labeling is called a difference cordial graph. In [3, 4, 5, 6] difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, $W_n \odot K_2$, $P_m \odot P_n$ and some more standard graphs have been investigated. In this paper, we investigate the difference cordial behaviour of $S(T_n)$, $S(A(T_n))$, $S(DA(T_n))$,

$S(Q_n)$, $S(A(Q_n))$, $S(DA(Q_n))$. Terms not defined here are used in the sense of Harary [2].

2 Methodology

In this article we use some number theory techniques for proving the main results.

3 Main Results

The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3 . $S(T_2) \cong C_6$ and hence difference cordial [3].

Theorem 3.1 $S(T_n)$ is difference cordial for all $n > 2$.

Proof. Let P_n be the path $u_1 u_2 \dots u_n$. Let $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$. Let $V(S(T_n)) = \{x_i, y_i, w_i : 1 \leq i \leq n-1\} \cup V(T_n)$ and $E(S(T_n)) = \{u_i x_i, x_i v_i, y_i v_i, y_i u_{i+1}, u_i w_i, w_i u_{i+1} : 1 \leq i \leq n-1\}$. Define an injective map $f : V(S(T_n)) \rightarrow \{1, 2, \dots, 5n-4\}$ by

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(w_i) &= 2i & 1 \leq i \leq n-1 \\ f(x_i) &= 2n+2i-2 & 1 \leq i \leq n-1 \\ f(y_i) &= 4n-3+i & 1 \leq i \leq n-1 \\ f(v_i) &= 2n+2i-1 & 1 \leq i \leq n-1. \end{aligned}$$

Since $e_f(0) = e_f(1) = 3n-3$, f is a difference cordial labeling of $S(T_n)$. ■

The Quadrilateral snake Q_n is obtained from the path P_n by replacing each edge of the path by a cycle C_4 .

Theorem 3.2 $S(Q_n)$ is difference cordial.

Proof. Let P_n be the path $u_1 u_2 \dots u_n$ and Let $V(Q_n) = \{v_i, w_i : 1 \leq i \leq n-1\} \cup V(P_n)$. Let $V(S(Q_n)) = \{x_i, u'_i, z_i, y_i : 1 \leq i \leq n-1\} \cup V(Q_n)$ and $E(S(Q_n)) = \{u_i u'_i, u'_i u_{i+1}, y_i u_{i+1} : 1 \leq i \leq n-1\} \cup$

$\{u_i x_i, x_i v_i, v_i z_i, z_i w_i, w_i y_i : 1 \leq i \leq n-1\}$. Define a map $f : V(S(Q_n)) \rightarrow \{1, 2, \dots, 7n-6\}$ by

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n-1 \\ f(v_i) &= 2n+3i-3 & 1 \leq i \leq n-1 \\ f(z_i) &= 2n+3i-2 & 1 \leq i \leq n-1 \\ f(w_i) &= 2n+3i-1 & 1 \leq i \leq n-1 \\ f(x_i) &= 5n-4+i & 1 \leq i \leq n-1 \\ f(y_i) &= 6n-5+i & 1 \leq i \leq n-1. \end{aligned}$$

Since $e_f(0) = e_f(1) = 4n-4$, f is a difference cordial labeling of $S(Q_n)$. ■

A double triangular snake DT_n consists of two triangular snakes that have a common path.

Theorem 3.3 $S(DT_n)$ is difference cordial.

Proof. Let $V(S(DT_n)) = \{u_i : 1 \leq i \leq n\} \cup \{x_i, y_i, v_i, x'_i, y'_i, w_i, z_i : 1 \leq i \leq n-1\}$ and $E(S(DT_n)) = \{u_i z_i, z_i u_{i+1}, u_i x_i, y_i u_{i+1}, x_i v_i, u_i x'_i : 1 \leq i \leq n-1\} \cup \{v_i y_i, w_i y'_i, x'_i w_i, y'_i u_{i+1} : 1 \leq i \leq n-1\}$. Define a map $f : V(S(DT_n)) \rightarrow \{1, 2, \dots, 8n-7\}$ by

$$\begin{aligned} f(u_i) &= 4i-3 & 1 \leq i \leq n \\ f(z_i) &= 7n-6+i & 1 \leq i \leq n-1 \\ f(x_i) &= 4i-2 & 1 \leq i \leq n-1 \\ f(v_i) &= 4i-1 & 1 \leq i \leq n-1 \\ f(y_i) &= 4i & 1 \leq i \leq n-1 \\ f(x'_i) &= 4n+2i-4 & 1 \leq i \leq n-1 \\ f(w_i) &= 4n+2i-3 & 1 \leq i \leq n-1 \\ f(y'_i) &= 6n-5+i & 1 \leq i \leq n-1. \end{aligned}$$

Obviously the above vertex labeling is a difference cordial labeling of $S(DT_n)$. ■

A double quadrilateral snake DQ_n consists of two triangular snakes that have a common path.

Theorem 3.4 $S(DQ_n)$ is difference cordial.

Proof. Let $V(S(DQ_n)) = V(DQ_n) \cup \{u'_i, v'_i, w'_i, z'_i, x'_i, y'_i, z_i : 1 \leq i \leq n-1\}$ and $E(S(DQ_n)) = \{u_i u'_i, u'_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v'_i, u_i x'_i, v'_i v_i, x'_i x_i, v_i z'_i, x_i z_i, z'_i w_i : 1 \leq i \leq n-1\} \cup \{z_i y_i, w_i w'_i, y_i y'_i, w'_i u_{i+1}, y'_i u_{i+1} : 1 \leq i \leq n-1\}$. Define a map $f : V(S(DQ_n)) \rightarrow \{1, 2, \dots, 12n-11\}$

$$\begin{aligned} f(u_i) &= 6i-5 & 1 \leq i \leq n \\ f(u'_i) &= 6i-4 & 1 \leq i \leq n-1 \\ f(v_i) &= 6i-3 & 1 \leq i \leq n-1 \\ f(z'_i) &= 6i-2 & 1 \leq i \leq n-1 \\ f(w_i) &= 6i-1 & 1 \leq i \leq n-1 \\ f(w'_i) &= 6i & 1 \leq i \leq n-1 \\ f(x'_i) &= 6n+2i-6 & 1 \leq i \leq n-1 \\ f(x_i) &= 6n+2i-5 & 1 \leq i \leq n-1 \\ f(z_i) &= 8n+i-7 & 1 \leq i \leq n-1 \\ f(y_i) &= 9n+i-8 & 1 \leq i \leq n-1 \\ f(y'_i) &= 10n+i-9 & 1 \leq i \leq n-1 \\ f(u'_i) &= 11n+i-10 & 1 \leq i \leq n-1. \end{aligned}$$

Since $e_f(0) = e_f(1) = 7n-7$, f is a difference cordial labeling of $S(DQ_n)$. ■

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1 u_2 \dots, u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i . That is every alternate edge of a path is replaced by C_3 .

Theorem 3.5 $S(A(T_n))$ is difference cordial.

Proof. Let the edges $u_i u_{i+1}, u_i v_i$ and $v_i u_{i+1}$ be subdivided by u'_i, x_i and y_i respectively.

Case 1. Let the triangle be starts from u_1 and ends with u_n .

In this case $S(A(T_n))$ consists of $\frac{7n-2}{2}$ vertices and $4n-2$ edges. Define a map $f : V(S(A(T_n))) \rightarrow \{1, 2, \dots, \frac{7n-2}{2}\}$ by

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n-1 \\ f(x_i) &= 2n-1+i & 1 \leq i \leq \frac{n}{2} \\ f(y_i) &= \frac{5n-2}{2} + i & 1 \leq i \leq \frac{n}{2} \\ f(v_{\frac{n}{2}-i+1}) &= 3n-1+i & 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 2n-1$, f is a difference cordial labeling of $S(A(T_n))$.

Case 2. Let the triangle be starts from u_2 and ends with u_{n-1} .

In this case, $S(A(T_n))$ has $\frac{7n-8}{2}$ vertices and $4n-6$ edges. Define an injective map $f : V(S(A(T_n))) \rightarrow \{1, 2, \dots, \frac{7n-8}{2}\}$ by

$$\begin{aligned} f(u_i) &= 2i-2 & 2 \leq i \leq n \\ f(u'_i) &= 2i-1 & 1 \leq i \leq n-1 \\ f(x_i) &= 2n-2+i & 1 \leq i \leq \frac{n-2}{2} \\ f(y_i) &= \frac{5n-6}{2} + i & 1 \leq i \leq \frac{n-2}{2} \\ f(v_i) &= 3n-3+i & 1 \leq i \leq \frac{n-2}{2}. \end{aligned}$$

$f(u_1) = 3n - 3$. Obviously, f is a difference cordial labeling of $S(A(T_n))$.

Case 3. Let the triangle be starts from u_2 and ends with u_n .

In this case, the order and size of $S(A(T_n))$ are $\frac{7n-5}{2}$ and $4n - 4$ respectively. The difference cordial labeling of $S(A(T_3))$ is given in figure 1.

Figure 1

For $n > 3$, Define an injective map $f : V(S(A(T_n))) \rightarrow \{1, 2, \dots, \frac{7n-5}{2}\}$ by

$$\begin{aligned} f(u_i) &= 2i - 1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n - 1 \\ f(x_i) &= 2n - 1 + i & 1 \leq i \leq \frac{n-1}{2} \\ f(y_i) &= \frac{5n-3}{2} + i & 1 \leq i \leq \frac{n-1}{2} \\ f(v_i) &= 3n - 2 + i & 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 2n - 2$, f is a difference cordial labeling of $S(A(T_n))$. ■

A double alternate triangular snake $DA(T_n)$ consist of two triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i .

Theorem 3.6 $S(DA(T_n))$ is difference cordial.

Proof. Let the edges u_iu_{i+1} , u_iv_i , v_iu_{i+1} , u_iw_i , w_iu_{i+1} be subdivided by u'_i , x_i , y_i , x'_i , y'_i respectively.

Case 1. Let the two triangles be starts from u_1 and ends with u_n .

Here, the number of vertices and edges in $S(DA(T_n))$ are $5n - 1$ and $6n - 2$ respectively. Define a map $f : V(S(DA(T_n))) \rightarrow \{1, 2, \dots, 5n - 1\}$ by

$$\begin{aligned} f(u_i) &= 2i - 1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n - 1 \\ f(x_i) &= 2n - 2 + 2i & 1 \leq i \leq \frac{n}{2} \\ f(v_i) &= 2n - 1 + 2i & 1 \leq i \leq \frac{n}{2} \\ f(y_{\frac{n}{2}-i+1}) &= 3n - 1 + i & 1 \leq i \leq \frac{n}{2} \\ f(x'_i) &= \frac{7n-2}{2} + i & 1 \leq i \leq \frac{n}{2} \\ f(y'_i) &= 4n - 1 + i & 1 \leq i \leq \frac{n}{2} \\ f(w_i) &= \frac{9n-2}{2} + i & 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 3n - 1$, f is a difference cordial labeling of $S(DA(T_n))$.

Case 2. Let the two triangles be starts from u_2 and ends with u_{n-1} .

In this case, the order and size of $S(DA(T_n))$ are $5n - 7$ and $6n - 10$ respectively. Define a function $f : V(S(DA(T_n))) \rightarrow \{1, 2, \dots, 5n - 7\}$ by $f(u_1) = 2n - 1$,

$$\begin{aligned} f(u_i) &= 2i - 2 & 2 \leq i \leq n \\ f(u'_i) &= 2i - 1 & 1 \leq i \leq n - 1 \\ f(x_i) &= 2n - 2 + 2i & 1 \leq i \leq \frac{n-2}{2} \\ f(v_i) &= 2n - 1 + 2i & 1 \leq i \leq \frac{n-2}{2} \\ f(y_i) &= 4n - 5 + i & 1 \leq i \leq \frac{n-2}{2} \\ f(x'_i) &= 3n + 2i - 4 & 1 \leq i \leq \frac{n-2}{2} \\ f(y'_i) &= \frac{9n-12}{2} + i & 1 \leq i \leq \frac{n-2}{2} \\ f(w_i) &= 3n + 2i - 3 & 1 \leq i \leq \frac{n-2}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 3n - 5$, f is a difference cordial labeling of $S(DA(T_n))$.

Case 3. Let the two triangles be starts from u_2 and ends with u_n .

In this case, the order and size of $S(DA(T_n))$ consists of $5n - 4$ vertices and $6n - 6$ edges respectively. Define a function $f : V(S(DA(T_n))) \rightarrow \{1, 2, \dots, 5n - 4\}$ by

$$\begin{aligned} f(u_i) &= 2i - 1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n - 1 \\ f(x_i) &= 2n + 2i - 2 & 1 \leq i \leq \frac{n-1}{2} \\ f(v_i) &= 2n + 2i - 1 & 1 \leq i \leq \frac{n-1}{2} \\ f(x'_i) &= 3n + 2i - 3 & 1 \leq i \leq \frac{n-1}{2} \\ f(w_i) &= 3n + 2i - 2 & 1 \leq i \leq \frac{n-1}{2} \\ f(y_i) &= 4n - 3 + i & 1 \leq i \leq \frac{n-1}{2} \\ f(y'_i) &= \frac{9n-7}{2} + i & 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 3n - 3$, f is a difference cordial labeling of $S(DA(T_n))$. ■

An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $u_1u_2 \dots u_n$ by joining u_i , u_{i+1} (alternatively) to new vertices v_i , w_i respectively and then joining v_i and w_i . That is every alternate edge of a path is replaced by a cycle C_4 .

Theorem 3.7 $S(AQ_n)$ is difference cordial.

Proof. Let the edges u_iu_{i+1} , u_iv_i , v_iw_i , w_iu_{i+1} be subdivided by u'_i , v'_i , z_i , w'_i respectively.

Case 1. Let the squares be starts from u_1 and ends with u_n .

In this case the order and size of $S(AQ_n)$ are $\frac{9n-2}{2}$ and $5n - 2$ respectively. Define a map $f : V(S(AQ_n)) \rightarrow$

$\{1, 2, \dots, \frac{9n-2}{2}\}$ as follows:

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n-1 \\ f(v'_i) &= 2n+2i-2 & 1 \leq i \leq \frac{n}{2} \\ f(v_i) &= 2n+2i-1 & 1 \leq i \leq \frac{n}{2} \\ f(w'_i) &= 3n-1+i & 1 \leq i \leq \frac{n}{2} \\ f(z_i) &= \frac{7n-2}{2}+i & 1 \leq i \leq \frac{n}{2} \\ f(w_{\frac{n}{2}-i+1}) &= 4n-1+i & 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = \frac{5n-2}{2}$, f is a difference cordial labeling of $S(AQ_n)$.

Case 2. Let the squares be starts from u_2 and ends with u_{n-1} .

The difference cordial labeling of $S(AQ_4)$ is given in figure 2.

Figure 2

For $n > 4$, Define a map $f : V(S(AQ_n)) \rightarrow \{1, 2, \dots, \frac{9n-12}{2}\}$ by

$$\begin{aligned} f(u_i) &= 2i-2 & 2 \leq i \leq n \\ f(u'_i) &= 2i-1 & 1 \leq i \leq n-1 \\ f(v'_i) &= 2n+2i-3 & 1 \leq i \leq \frac{n-2}{2} \\ f(v_i) &= 2n+2i-2 & 1 \leq i \leq \frac{n-2}{2} \\ f(w'_i) &= 3n-4+i & 1 \leq i \leq \frac{n-2}{2} \\ f(z_i) &= \frac{7n-10}{2}+i & 1 \leq i \leq \frac{n-2}{2} \\ f(w_i) &= 4n-6+i & 1 \leq i \leq \frac{n-2}{2} \end{aligned}$$

and $f(u_1) = \frac{9n-12}{2}$. Since $e_f(0) = e_f(1) = 5n-8$, f is a difference cordial labeling of $S(AQ_n)$.

Case 3. Let the squares be starts from u_2 and ends with u_n .

The difference cordial labeling of $S(AQ_3)$ is given in figure 3.

Figure 3

For $n > 3$, Define a map $f : V(S(AQ_n)) \rightarrow \{1, 2, \dots, \frac{9n-7}{2}\}$ by

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n-1 \\ f(v'_i) &= 2n+2i-2 & 1 \leq i \leq \frac{n-1}{2} \\ f(v_i) &= 2n+2i-1 & 1 \leq i \leq \frac{n-1}{2} \\ f(z_i) &= 3n-2+i & 1 \leq i \leq \frac{n-1}{2} \\ f(w_i) &= \frac{7n-5}{2}+i & 1 \leq i \leq \frac{n-1}{2} \\ f(w'_i) &= 4n-3+i & 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = \frac{5n-5}{2}$, f is a difference cordial labeling of $S(AQ_n)$. ■

Illustration 1 The difference cordial labeling of $DA(Q_8)$ is given in figure 4.

Figure 4

The difference cordial labeling of $DA(Q_8)$ is given in figure 5.

Figure 5

The difference cordial labeling of $DA(Q_7)$ is given in figure 6.

Figure 6

A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i .

Theorem 3.8 $S(DAQ_n)$ is difference cordial.

Proof. Let the edges $u_iu_{i+1}, u'_iv_i, v_iw_i, w_iu'_{i+1}, u'_ix_i, x_iy_i, y_iu_{i+1}$ be subdivided by $u_i, v_i, z_i, w_i, x_i, z_i, y_i$ respectively.

Case 1. Let the squares be starts from u_1 and ends with u_n .

The order and size of $S(DAQ_n)$ are $7n-1$ and $8n-2$

respectively. Define a map $f : V(S(DAQ_n)) \rightarrow \{1, 2, \dots, 7n-1\}$ by

$$\begin{aligned} f(u_i) &= 2i-1 & 1 \leq i \leq n \\ f(u'_i) &= 2i & 1 \leq i \leq n-1 \\ f(v_i) &= 2n+3i-3 & 1 \leq i \leq \frac{n}{2} \\ f(z_i) &= 2n+3i-2 & 1 \leq i \leq \frac{n}{2} \\ f(w_i) &= 2n+3i-1 & 1 \leq i \leq \frac{n}{2} \\ f(x_i) &= \frac{7n-6}{2} + 3i & 1 \leq i \leq \frac{n}{2} \\ f(z'_i) &= \frac{7n-4}{2} + 3i & 1 \leq i \leq \frac{n}{2} \\ f(y_i) &= \frac{7n-2}{2} + 3i & 1 \leq i \leq \frac{n}{2} \\ f(x'_i) &= \frac{11n-2}{2} + i & 1 \leq i \leq \frac{n}{2} \\ f(w'_i) &= \frac{13n-2}{2} + i & 1 \leq i \leq \frac{n}{2} \\ f(y'_{\frac{n}{2}-i+1}) &= 5n-1+i & 1 \leq i \leq \frac{n}{2} \\ f(v'_i) &= 6n-1+i & 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 4n-1$, f is a difference cordial labeling of $S(DAQ_n)$.

Case 2. Let the squares be starts from u_2 and ends with u_{n-1} .

The order and size of $S(DAQ_n)$ are $7n-11$ and $8n-14$ respectively. Define a map $f : V(S(DAQ_n)) \rightarrow \{1, 2, \dots, 7n-11\}$ by

$$\begin{aligned} f(u_i) &= 2i-2 & 2 \leq i \leq n \\ f(u'_i) &= 2i-1 & 1 \leq i \leq n-1 \\ f(v_i) &= 2n+3i-4 & 1 \leq i \leq \frac{n-2}{2} \\ f(z_i) &= 2n+3i-3 & 1 \leq i \leq \frac{n-2}{2} \\ f(w_i) &= 2n+3i-2 & 1 \leq i \leq \frac{n-2}{2} \\ f(x_i) &= \frac{7n-14}{2} + 3i & 1 \leq i \leq \frac{n-2}{2} \\ f(z'_i) &= \frac{7n-12}{2} + 3i & 1 \leq i \leq \frac{n-2}{2} \\ f(y_i) &= \frac{7n-10}{2} + 3i & 1 \leq i \leq \frac{n-2}{2} \\ f(w'_i) &= \frac{11n-18}{2} + i & 1 \leq i \leq \frac{n-2}{2} \\ f(y'_i) &= \frac{13n-22}{2} + i & 1 \leq i \leq \frac{n-2}{2} \\ f(v'_i) &= 5n-8+i & 1 \leq i \leq \frac{n-2}{2} \\ f(x'_i) &= 6n-10+i & 1 \leq i \leq \frac{n-2}{2}. \end{aligned}$$

and $f(u_1) = 7n-11$. Since $e_f(0) = e_f(1) = 4n-7$, f is a difference cordial labeling of $S(DAQ_n)$.

Case 3. Let the squares be starts from u_2 and ends with u_n .

Assign the labels to the vertices u_i ($1 \leq i \leq n$), u'_i ($1 \leq i \leq n-1$), v_i, z_i, w_i ($1 \leq i \leq \frac{n-1}{2}$) as in case 1

$$\begin{aligned} f(x_i) &= \frac{7n-9}{2} + 3i & 1 \leq i \leq \frac{n-1}{2} \\ f(z'_i) &= \frac{7n-7}{2} + 3i & 1 \leq i \leq \frac{n-1}{2} \\ f(y_i) &= \frac{7n-5}{2} + 3i & 1 \leq i \leq \frac{n-1}{2} \\ f(w'_i) &= \frac{11n-9}{2} + i & 1 \leq i \leq \frac{n-1}{2} \\ f(y'_i) &= \frac{13n-11}{2} + i & 1 \leq i \leq \frac{n-1}{2} \\ f(v'_i) &= 5n-4+i & 1 \leq i \leq \frac{n-1}{2} \\ f(x'_i) &= 6n-5+i & 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

Since $e_f(0) = e_f(1) = 4n-4$, f is a difference cordial labeling of $S(DAQ_n)$. ■

4 Conclusion

In this paper we have proved that subdivision of quadrilateral snakes, triangular snakes, Alternate triangular snakes, double triangular snakes, Alternate quadrilateral snakes, Double quadrilateral snakes, Double alternate triangular snakes, double alternate quadrilateral snakes are difference cordial. Investigation of difference cordial labeling behavior of subdivision of $G_1 \cup G_2$, $G_1 + G_2$, $G_1 \times G_2$ etc are the open problems for future research.

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