Adaptive Switched Linear Systems with Average Dwell Time to Compensate External Disturbances

Leonardo Acho

CoDALab, Departament de Matemàtica Aplicada III,
Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona,
Universitat Politècnica de Catalunya,
Comte d'Urgell,187, 08036 Barcelona, Spain
*Corresponding Author:leonardo.acho@upc.edu

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Abstract Given a linear switched system composed of both Hurwitz stable and unstable subsystems, and with additive unknown constant disturbance, a switching law with average dwell time between Hurwitz stable and unstable subsystems is proposed, together with an adaptive algorithm to compensate the unknown additive disturbance. Using Lyapunov theory, exponential stability of a desired degree of the plant state is assured and bounded-ness of the adaptive dynamic too.

Keywords Adaptive control, Switched systems, Dwell-time

1 Introduction

There exist many practical problems where the available choice is to switch among a finite set of subsystems [1], [2], [3]. Also, in some practical problems [4], the only way to incorporate a logic-based decision system is by switching among a family of subsystems. Switched systems are of *variable structure* or *multi-modal* class [5]. According to [6], a switched system can be viewed as a hybrid dynamical system which is composed of a family of continuous-time subsystems along with a switching law among them.

In comparison to sliding mode systems (a type of variable structure one), which is an approach recognized as an efficient tool to design robust systems, switching systems present chattering behavior. This behavior, in some situations, would not be considered acceptable [7]. Although there exist solutions to reduce this phenomenon [8] (including the well known super-twisting algorithm [9]), dwell switching design can be viewed as an alternative tool of switching laws without chattering. Moreover, dwell average switching systems do not exhibit *Zenoness* and *beating* effects. These effects can

appear on hybrid dynamical systems [10].

Adaptive control provides adaptation to adjust a system with parametric uncertainties [11]. Motivated by the adaptive control law presented in [12] to face with additive constant disturbance, we propose a switching law with average dwell time along with an adaptive algorithm to compensate it. This scheme is applied to linear switched systems consisting of both Hurwitz stable and unstable subsystems. Exponential stability of a desired degree is guaranteed for the states of the plant. Bounded-ness of the adaptive dynamic is assured too. Although in [6] it is proposed a dwell average switching law for linear systems in the presence of a class of nonlinear additive perturbation satisfying a kind of Lipschitz condition, the constant additive case is not considered. This class of constant additive perturbation is presented in some engineering problems [12].

The remainder of this article is organized as follows. Section 2 gives the main theory and notation on switched systems with average dwell time. Section 3 states our main result. A numerical example supporting our theory is given in Section 4. Finally, Section 5 gives the conclusions.

2 Switching design with average dwell time

Consider the linear switched system with additive constant disturbance given by:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + d - \hat{\phi}(t), \quad x(t_0) = x_0, \quad (1)$$

where $x(t) \in R^n$ is the state, t_0 is the initial time, $d \in R^n$ is the additive constant disturbance, and x_0 is the initial state. The piecewise constant function of time $\sigma(t) : [t_0, \infty) \rightarrow I_N = \{1, 2, \dots, N\}$ is called a switching signal, and $A_{\sigma(t)} : [t_0, \infty) \rightarrow \{A_1, A_2, \dots, A_N\}$ represents a piecewise constant function of matrices A_i ($i \in I_N$). See Figure 1.

Figure 1. Schematic representation of system (1).

The matrices A_i ($i \in I_N$) are assumed of appropriate dimension, and $N > 1$ represents the number of subsystems. It is also assumed the switched system (1) is composed of both Hurwitz stable and unstable subsystems.

The control objective consists to find a switching law with *average dwell time*, along with a dynamic estimation $\hat{\phi}(t)$ to d , such that the plant states goes to the equilibrium point as time goes to infinity *exponentially with a stability degree*, while keeping bounded the estimation dynamic $\hat{\phi}(t)$ for all $t \geq t_0$ (i.e., there exist a constant value v such that $\|\hat{\phi}(t)\| \leq v, \forall t \geq t_0$).

Given a switching signal $\sigma(t)$ and any $t > \tau \geq 0$, the number of switchings of $\sigma(t)$ on the interval (τ, t) , denoted by $N_\sigma(\tau, t)$, satisfies:

$$N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_a}, \quad (2)$$

for a given constants N_0 and τ_a . The constant N_0 is named after *the chatter bound*, and τ_a is called the *average dwell constant time* [4], [6]. Basically, there may exist consecutive switchings separated by less than τ_a , but the average time interval between consecutive switchings is not less than τ_a [6]¹. Let us introduce the notation $S_a[\tau_a, N_0]$ to denote the set of all switching signals satisfying (2).

Definition 1 [4], [6]. *The state of the switched system (1) is said to be globally exponentially stable with stability degree $\lambda \geq 0$ if $\|x(t)\| \leq e^{\alpha - \lambda(t-t_0)} \|x_0\|$ holds for all $t \geq t_0$ and a constant α .*

For simplicity, suppose that matrices $A_1, \dots, A_r$ ($1 \leq r < N$) are unstable Hurwitz subsystems and the remaining matrices are the Hurwitz stable ones. Then there exist a set of positive scalars $\lambda_1, \lambda_2, \dots, \lambda_N$ such that $A_i - \lambda_i I$ ($i \leq r$) and $A_i + \lambda_i I$ ($i > r$) are Hurwitz stable matrices satisfying the following Lyapunov equations [4], [6]:

$$(A_i - \lambda_i I)^T P_i + P_i (A_i - \lambda_i I) < 0, \quad i \leq r, \quad (3)$$

and

$$(A_i + \lambda_i I)^T P_i + P_i (A_i + \lambda_i I) < 0, \quad i > r, \quad (4)$$

where the matrices P_i 's are symmetric and positive definite.

¹A special case deserves attention [6]. If $N_0 = 0$, then (2) states that the average time interval between consecutive switchings is at least τ_a .

So, for any piecewise switching signal $\sigma(t)$, let us use the notation $T^+(t)$ and $T^-(t)$ to denote the total activation time of Hurwitz unstable subsystems and Hurwitz stable subsystems, respectively, during the time interval $[t_0, t)$. Define $\lambda^+ = \max_{i \leq r} \lambda_i$ and $\lambda^- = \min_{i > r} \lambda_i$. Then, for any given $\lambda \in (0, \lambda^-)$, there exists $\lambda^* \in (\lambda, \lambda^-)$, where λ^* is chosen arbitrarily [6].

3 Main result

To begin with the main result, consider the next switching law:

(S₁) **Switching law** [6]: *Determine the switching signal $\sigma(t)$ so that*

$$\inf_{t \geq t_0} \frac{T^-(t)}{T^+(t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*}, \quad (5)$$

holds for any given initial time t_0 .

Theorem 1 *Under the switching law (S₁), there is a finite constant τ_a^* such that the switched system (1), $x(t)$ is globally exponentially stable with stability degree λ over $S_a[\tau_a, N_0]$, for any average dwell time $\tau_a \geq \tau_a^*$, any chatter bound $N_0 \geq 0$, and with $\hat{\phi}(t)$ a solution to:*

$$\dot{\hat{\phi}}(t) = \beta P_{\sigma(t)} x(t), \quad (6)$$

where β is a given positive constant and $P_{\sigma(t)}$ are the corresponding solutions to the Lyapunov equations (3) and (4), applied over the switching law (S₁). Moreover, $\hat{\phi}(t)$ remains bounded (i.e., there exist a constant value v such that $\|\hat{\phi}(t)\| \leq v \quad \forall t \geq t_0$). Furthermore, if:

$$\mu = \sup_{k, i \in I_N} \frac{\lambda_M(P_k)}{\lambda_m(P_i)}, \quad (7)$$

where $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ denote the largest and smallest eigenvalues of a symmetric matrix, then

$$\tau_a^* = \frac{\ln(\mu)}{2(\lambda^+ - \lambda)}. \quad (8)$$

Proof. Similarly to the proof of (Theorem 1, [6]) but using the Lyapunov function:

$$V(x, \hat{\phi}, t) = V_a(x, t) + (d - \hat{\phi}(t))^T \beta^{-1} (d - \hat{\phi}(t)), \quad (9)$$

with ([6]):

$$V_a(x, t) = x^T P_{\sigma(t)} x. \quad (10)$$

A sketch is as follows.

1) Each V_i in (9) is continuous and its time derivative along the solutions of the corresponding subsystems and (6), satisfies:

$$\dot{V}_i \leq \begin{cases} 2\lambda_i V_{a_i}, & i \leq r \\ -2\lambda_i V_{a_i}, & i > r \end{cases}. \quad (11)$$

2) There exist constant scalars $\alpha_2 \geq \alpha_1 > 0$ such that:

$$\alpha_1 \|x(t)\|^2 \leq V_{a_i}(x, t) \leq \alpha_2 \|x(t)\|^2, \quad (12)$$

$\forall x \in R^n$ and $\forall i \in I_N$.

3) There exists a constant scalar $\mu \geq 1$ such that:

$$V_{a_i}(x, t) \leq \mu V_{a_j}(x, t), \quad (13)$$

$\forall x \in R^n$ and $\forall i, j \in I_N$.

For a discussion on properties 2) and 3), see [6] (in fact, a value for μ is given by (7) [6]).

So, for any piecewise constant switching signal $\sigma(t)$, and any $t > t_0$, let $t_1 < t_2 < \dots < t_{N_{\sigma(t_0, t)}}$ be the switching points of $\sigma(t)$ over the time interval (t_0, t) , and suppose that the p -th subsystem is activated during $[t_i, t_{i+1})$ [6]. Then, for any $\zeta \in [t_i, t_{i+1})$, and from (11), we have:

$$V(\zeta) \leq \begin{cases} e^{2\lambda^+(\zeta-t_i)} V_a(t_i), & p_i \leq r \\ e^{-2\lambda^-(\zeta-t_i)} V_a(t_i), & p_i > r \end{cases} \quad (14)$$

According to [6], and under the Theorem 1 conditions, the above means that:

$$\|x(t)\| \leq \sqrt{\left(\frac{\alpha_2}{\alpha_1}\right)} e^{\alpha-\lambda(t-t_0)} \|x_0\|, \quad (15)$$

where $\alpha = \frac{N_0 \ln(\mu)}{2}$ with $N_0 \geq 0$ being an arbitrary value. This implies that the plant state $(x(t))$ converges exponentially with a prescribed degree. Using this fact, and from (11), we conclude that $\dot{V}(t) \leq 0$ meaning that $\hat{\phi}(t)$ is bounded. This conclude the sketch of our proof. $\diamond$

4 Numerical example

Consider the switched system (1), given in [6] but the additive constant disturbance, with:

$$A_1 = \begin{bmatrix} -9 & 10 \\ -10 & 11 \end{bmatrix}, A_2 = \begin{bmatrix} -20 & 10 \\ 10 & -20 \end{bmatrix}, \quad (16)$$

and

$$d = \begin{bmatrix} 10 \\ -10 \end{bmatrix}. \quad (17)$$

Here, A_1 is the Hurwitz unstable subsystem with $\lambda(A_1) = \{1, 1\}$, and A_2 is the Hurwitz stable one with $\lambda(A_2) = \{-10, -30\}$. Using $\lambda_1 = 10$ and $\lambda_2 = 9$, the solution to the Lyapunov equations (3) and (4), yields [6]:

$$P_1 = \begin{bmatrix} 0.45 & -0.05 \\ -0.05 & 0.45 \end{bmatrix}, P_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}. \quad (18)$$

Then, with $\lambda^+ = \lambda_1$ and $\lambda^- = \lambda_2$, we can choose $\lambda = 1$ and $\lambda^* = 6$. So, the switching law (S_1) gives:

$$\frac{T^-(t)}{T^+(t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} = \frac{16}{3}, \quad (19)$$

and

$$\tau_a \geq \tau_a^* = \frac{\ln(\mu)}{2(\lambda^* - \lambda)} = 0.041. \quad (20)$$

According to *Theorem 1*, a switching law with average dwell time of 0.041 seconds can be realized by activating A_1 and A_2 (for the switched system part), and P_1 and P_2 (for the adaptive system (6)), alternatively with their activation time of 0.015 and 0.08 seconds, respectively. This commutation law satisfies both, the switching activation law and the average dwell time. Using $\beta = 100$, $x_0 = [x_1(0), x_2(0)]^T = [10, 60]^T$, and $\hat{\phi}(0) = [\hat{\phi}_1(0), \hat{\phi}_2(0)]^T = [0, 0]^T$, experimental simulation results are shown in Figures 2 and 3. Exponential

stability of the plant states $(x_1(t))$ and $x_2(t))$ are appreciated, and the estimation to the vector $d = [10, -10]^T$ is good. Figures 4 and 5 show the case when the disturbance belongs to a class of *slowly* time varying scenario $d = [10 \sin(t), -10]^T$.

Figure 2. Numerical results: evolution of the disturbance estimations over time.

Figure 3. Numerical results: evolution of the plan states over time.

Figure 4. Numerical results: evolution of the plan states for the *slow* time varying disturbance case.

5 Conclusions

Essentially, in this paper, we extended the results presented in [6]. This extension involves the inclusion of additive constant perturbation to the switched system instead of dealing with additive nonlinear perturbation [6]. According to the given numerical experiments, for the *slow* time varying disturbance case, our propose goes well too.

Figure 5. Numerical results: time evolution of the dynamic estimations for the *slow* time varying disturbance case.

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