Approximating Common Fixed Points of Two Sequences of Uniformly Quasi-lipschitzian Mappings in Convex Cone Metric Spaces

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Abstract A convex cone metric space is a cone metric space with a convex structure. In this paper, we extend an Ishikawa type iterative scheme with errors to approximate a common fixed point of two sequences of uniformly quasi-Lipschitzian mappings to convex cone metric spaces. Our result generalizes Theorem 2 in [1].

Keywords Convex Structure, Convex Cone Metric Spaces, Ishikawa Type Iterative Scheme, Common Fixed Point, Uniformly Quasi-lipschitzian Mappings, Asymptotically Quasi-nonexpansive Mappings, Asymptotically Non-expansive Mappings

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1. Introduction and Preliminaries

It is desirable to add the convex structure in [3, 6] to cone metric spaces and consider an Ishikawa type iterative scheme with errors to approximate a common fixed point of two sequences of uniformly quasi-Lipschitzian mappings in convex cone metric spaces.

Throughout this paper, E is a normed vector space with a normal solid cone P .

Definition 1.1. [2] A nonempty subset P of E is called a cone if P is closed, $P \neq \{\theta\}$, for $a, b \in \mathbb{R}^+ = [0, \infty)$ and $x, y \in P$, $ax + by \in P$ and $P \cap \{-P\} = \{\theta\}$. We define a partial ordering $\preceq$ in E as $x \preceq y$ if $y - x \in P$. $x \ll y$ indicates that $y - x \in \text{int}P$ and $x \prec y$ means that $x \preceq y$ but $x \neq y$. A cone P is said to be solid if its interior $\text{int}P$ is nonempty. A cone P is said to be normal if there exists a positive number k such that for $x, y \in P$, $0 \preceq x \preceq y$ implies $\|x\| \leq k\|y\|$. The least positive number k is called the normal constant of P .

Definition 1.2. [2] Let X be a nonempty set. A mapping $d : X \times X \rightarrow (E, P)$ is called a cone metric if (i) for $x, y \in X$, $0 \preceq d(x, y)$ and $d(x, y) = \theta$ iff $x = y$, (ii) for $x, y \in X$, $d(x, y) = d(y, x)$ and (iii) for $x, y, z \in X$, $d(x, y) \preceq d(x, z) + d(z, y)$. A nonempty set X with a cone metric $d : X \times X \rightarrow (E, P)$ is called a cone metric space denoted by (X, d) , where P is a solid normal cone.

Definition 1.3. [5] Let (X, d) be a cone metric space and $T : (X, d) \rightarrow (X, d)$ a mapping.

- (i) T is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\}$ in $[1, \infty)$ such that $\lim_{n \rightarrow \infty} k_n = 1$ with $d(T^n x, T^n y) \preceq k_n \cdot d(x, y)$ for all $x, y \in X$ ($n \in \mathbb{N} \cup \{0\}$).
- (ii) T is said to be asymptotically quasi-nonexpansive if there exists a sequence $\{k_n\}$ in $[1, \infty)$ such that $\lim_{n \rightarrow \infty} k_n = 1$ with $d(T^n x, p) \preceq k_n \cdot d(x, p)$ for all $x \in X$ and $p \in F(T)$, the set of fixed points of T ($n \in \mathbb{N} \cup \{0\}$).
- (iii) T is said to be uniformly quasi-Lipschitzian if there exists $L > 0$ such that $d(T^n x, p) \preceq L \cdot d(x, p)$ for all $x \in X$ and $p \in F(T)$ ($n \in \mathbb{N} \cup \{0\}$).

Remark 1.1. If $F(T) \neq \emptyset$, then (i) implies (ii), and (ii) implies (iii) by putting $L = \sup_{n \geq 0} k_n < \infty$, but not conversely from some examples shown in metric spaces [8].

Now we consider the convex structure in cone metric spaces.

Definition 1.4. Let (X, d) be a cone metric space. A mapping $W : X^3 \times I^3 \rightarrow X$ is called a convex structure on X if $d(W(x, y, z, a_n, b_n, c_n), u) \preceq a_n \cdot d(x, u) + b_n \cdot d(y, u) + c_n \cdot d(z, u)$ for real sequences $\{a_n\}, \{b_n\}$ and $\{c_n\}$ in $I = [0, 1]$ satisfying $a_n + b_n + c_n = 1$ ($n \in \mathbb{N}$) and x, y, z and $u \in X$. A cone metric space (X, d) with a convex structure W is called a convex cone metric space and denoted as (X, d, W) . A nonempty subset C of a convex cone metric space (X, d, W) is said to be convex if $W(x, y, z, a, b, c) \in C$ for all $x, y, z \in C$ and $a, b, c \in I$.

Definition 1.5. Let (X, d) be a cone metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

- (i) If for any $c \in E$ with $0 \ll c$, there exists a natural number N such that for all $n > N$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to converge to x and denoted as $\lim_{n \rightarrow \infty} x_n = x$ or $x_n \rightarrow x$ (as $n \rightarrow \infty$).
- (ii) If for any $c \in E$ with $0 \ll c$, there exists a natural number N such that for all $n, m > N$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X .
- (iii) (X, d) is said to be complete if every Cauchy sequence converges.

Proposition 1.1. [2] Let $\{x_n\}$ be a sequence in a cone metric space (X, d) and P be a normal cone. Then

- (i) $\{x_n\}$ converges to x in X if and only if $d(x_n, x) \rightarrow 0$ (as $n \rightarrow \infty$) in E .
- (ii) $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \rightarrow 0$ (as $n, m \rightarrow \infty$) in E .

Let C be a nonempty convex subset of a convex cone metric space (X, d, W) . Let $S_i, T_i : C \rightarrow C$ be uniformly quasi-Lipschitzian mappings with Lipschitzian constants $L_i > 0$ and $L'_i > 0$, respectively ($i \in \mathbb{N}$).

Let $\{u_n\}$ and $\{v_n\}$ be sequences in C and $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}$ and $\{c'_n\}$ be sequences in $[0, 1]$ with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ ($n \in \mathbb{N} \cup \{0\}$).

For any given $x_0 \in C$, a sequence $\{x_n\}$ defined as

$$\begin{cases} x_{n+1} = W(x_n, S_{n+1}^n y_n, u_n, a'_n, b'_n, c'_n), \\ y_n = W(x_n, T_{n+1}^n x_n, v_n, a_n, b_n, c_n), \end{cases} \quad (1.1)$$

is called the Ishikawa type iteration process with errors for two sequences $\{S_i\}_{i \in \mathbb{N}}$ and $\{T_i\}_{i \in \mathbb{N}}$.

In this paper, we present more generalized convergence theorem concerning Ishikawa type iterative schemes with errors for approximating a common fixed point of two sequences of uniformly quasi-Lipschitzian mappings in convex cone metric spaces by comparing the convergence theorems considered in convex metric spaces [5, 7, 8]. Our result generalizes Theorem 2 in [1].

2. Main Results

Lemma 2.1. [4] Let $\{p_n\}, \{q_n\}$ and $\{r_n\}$ be sequences of nonnegative real numbers with $\sum_{n=1}^{\infty} q_n < \infty$ and $\sum_{n=1}^{\infty} r_n < \infty$. If $p_{n+1} \leq (1 + q_n) \cdot p_n + r_n$ ($n \in \mathbb{N} \cup \{0\}$), then

- (i) $\lim_{n \rightarrow \infty} p_n$ exists, and
- (ii) If $\varliminf_{n \rightarrow \infty} p_n = 0$, then $\lim_{n \rightarrow \infty} p_n = 0$.

Theorem 2.2 Let $d : X \times X \rightarrow (E, P)$ be a cone metric, where P is a solid normal cone with the normal constant k . Let C be a nonempty closed convex subset of complete convex cone metric space (X, d, W) and $S_i, T_i : C \rightarrow C$ be uniformly quasi-Lipschitzian mappings with constants $L_i > 0$ and $L'_i > 0$, respectively ($i \in \mathbb{N}$). Suppose that $F := \left(\bigcap_{i=1}^{\infty} F(S_i) \right) \cap \left(\bigcap_{i=1}^{\infty} F(T_i) \right)$ is nonempty and bounded. Let $\{x_n\}$ be a sequence in C defined by (1.1) with bounded sequences $\{u_n\}$ and $\{v_n\}$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{a'_n\}, \{b'_n\}$ and $\{c'_n\}$ be real sequences in I with $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ ($n \in \mathbb{N} \cup \{0\}$) such that $\sum_{n=0}^{\infty} (b'_n + c'_n) < \infty$.

Assume that $\{L_i : i \in \mathbb{N}\}$ and $\{L'_i : i \in \mathbb{N}\}$ are bounded.

Then the followings are equivalent;

- (i) $\{x_n\}$ converges to a common fixed point $p \in F$,
- (ii) $\varliminf_{n \rightarrow \infty} d(x_n, F) = 0$, where $d(x, F) = \inf_{q \in F} d(x, q)$.

Proof. Obviously, (i) implies (ii). Now we consider that (ii) implies (i).

- (1) Firstly, we show that $\lim_{n \rightarrow \infty} d(x_n, F) = 0$.

For any $p \in F$, from (1.1) we have

$$\begin{aligned} d(x_{n+1}, p) &= d(W(x_n, S_{n+1}^n y_n, u_n, a'_n, b'_n, c'_n), p) \\ &\preceq a'_n \cdot d(x_n, p) + b'_n \cdot d(S_{n+1}^n y_n, p) + c'_n \cdot d(u_n, p) \\ &\preceq a'_n \cdot d(x_n, p) + b'_n \cdot L \cdot d(y_n, p) + c'_n \cdot d(u_n, p) \end{aligned}$$

and

$$\begin{aligned} d(y_n, p) &= d(W(x_n, T_{n+1}^n x_n, v_n, a_n, b_n, c_n), p) \\ &\preceq a_n \cdot d(x_n, p) + b_n \cdot d(T_{n+1}^n x_n, p) + c_n \cdot d(v_n, p) \\ &\preceq a_n \cdot d(x_n, p) + b_n \cdot L \cdot d(x_n, p) + c_n \cdot d(v_n, p) \\ &= (a_n + b_n \cdot L) \cdot d(x_n, p) + c_n \cdot d(v_n, p), \end{aligned}$$

where $L = \max \left\{ \sup_{i \in \mathbb{N}} L_i, \sup_{i \in \mathbb{N}} L'_i \right\}.$

So,

$$\begin{aligned} d(x_{n+1}, p) &\preceq a'_n \cdot d(x_n, p) + b'_n \cdot L \cdot [(a_n + b_n \cdot L) \cdot d(x_n, p) + c_n \cdot d(v_n, p)] + c'_n \cdot d(u_n, p) \\ &\preceq [1 + b'_n \cdot L \cdot (1 + L)] \cdot d(x_n, p) + c'_n \cdot d(u_n, p) + b'_n \cdot c_n \cdot L \cdot d(v_n, p) \\ &\preceq [1 + b'_n \cdot L \cdot (1 + L)] \cdot d(x_n, p) + [d(u_n, p) + L \cdot d(v_n, p)](b'_n + c'_n) \\ &= [1 + b'_n \cdot L \cdot (1 + L)] \cdot d(x_n, p) + \eta_n \cdot u, \end{aligned}$$

where $u = \sup_{p \in F, n \geq 1} \{d(u_n, p) + L \cdot d(v_n, p)\}$ and $\eta_n = b'_n + c'_n$ ($n \in \mathbb{N} \cup \{0\}$).

Thus, by the normality of P , for the normal constant $k > 0$

$$\|d(x_{n+1}, p)\| \leq k \cdot [1 + b'_n \cdot L \cdot (1 + L)] \cdot \|d(x_n, p)\| + k \cdot \|u\| \cdot \eta_n.$$

Since p is an arbitrary point in F ,

$$\|d(x_{n+1}, F)\| \leq k \cdot [1 + b'_n \cdot L \cdot (1 + L)] \cdot \|d(x_n, F)\| + k \cdot \|u\| \cdot \eta_n.$$

Since $\sum_{n=0}^{\infty} \eta_n = \sum_{n=0}^{\infty} (b'_n + c'_n)$ is finite, by Lemma 2.1, $\lim_{n \rightarrow \infty} \|d(x_n, F)\|$ exists, so $\lim_{n \rightarrow \infty} d(x_n, F)$ exists. Now $\varprojlim_{n \rightarrow \infty} d(x_n, F) = 0$ implies $\lim_{n \rightarrow \infty} d(x_n, F) = 0$.

(2) Secondly, we show that $\{x_n\}$ is a Cauchy sequence in C .

Since $\lim_{n \rightarrow \infty} \|d(x_n, F)\| = 0$ and $\sum_{n=0}^{\infty} \eta_n < \infty$, for any positive real number ε , there exists a natural number $N_0 \in \mathbb{N}$ such that $\|d(x_n, F)\| \leq \frac{\varepsilon}{4M_1}$ for $n \geq N_0$ and $\sum_{n=N_0}^{\infty} \eta_n \leq \frac{\varepsilon}{4\|u\| \cdot M_1}$,

where $M_1 = e^{L \cdot (1+L) \sum_{k=0}^{\infty} b'_k}.$

In particular, there exist a point $p_1 \in F$ and a positive integer $N_1 > N_0$ such that

$$\|d(x_{N_1}, p_1)\| \leq \frac{\varepsilon}{4M_1}.$$

On the other hand, by the fact that

$$\|d(x_{n+1}, p)\| \leq k \cdot (1 + b'_n \cdot L \cdot (1 + L)) \cdot \|d(x_n, p)\| + k \cdot \|u\| \cdot \eta_n$$

and the inequality $1 + x \leq e^x$ for $x \geq 0$, we have

$$\begin{aligned}
\|d(x_{n+m}, p)\| &\leq k_1 \cdot (1 + b'_{n+m-1} \cdot L \cdot (1 + L)) \cdot \|d(x_{n+m-1}, p)\| + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\leq k_1 \cdot e^{b'_{n+m-1} \cdot L \cdot (1+L)} \cdot \|d(x_{n+m-1}, p)\| + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\leq k_1 \cdot e^{b'_{n+m-1} \cdot L \cdot (1+L)} \cdot [k_2 \cdot (1 + b'_{n+m-2} \cdot L \cdot (1 + L)) \cdot \|d(x_{n+m-2}, p)\| \\
&\quad + k_2 \cdot \|u\| \cdot \eta_{n+m-2}] + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\leq k_1 \cdot e^{b'_{n+m-1} \cdot L \cdot (1+L)} [k_2 \cdot e^{b'_{n+m-2} \cdot L \cdot (1+L)} \cdot \|d(x_{n+m-2}, p)\| + k_2 \cdot \|u\| \cdot \eta_{n+m-2}] \\
&\quad + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\leq k_1 \cdot k_2 \cdot e^{(b'_{n+m-1} + b'_{n+m-2}) \cdot L \cdot (1+L)} \cdot [k_3 \cdot e^{b'_{n+m-3} \cdot L \cdot (1+L)} \|d(x_{n+m-3}, p)\| \\
&\quad + k_3 \cdot \|u\| \cdot \eta_{n+m-3}] + k_1 \cdot k_2 \cdot e^{b'_{n+m-1} \cdot L \cdot (1+L)} \cdot \|u\| \cdot \eta_{n+m-2} + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&= k_1 \cdot k_2 \cdot k_3 \cdot e^{(b'_{n+m-1} + b'_{n+m-2} + b'_{n+m-3}) \cdot L \cdot (1+L)} \|d(x_{n+m-3}, p)\| \\
&\quad + k_1 \cdot k_2 \cdot k_3 \cdot e^{(b'_{n+m-1} + b'_{n+m-2}) \cdot L \cdot (1+L)} \cdot \|u\| \cdot \eta_{n+m-3} \\
&\quad + k_1 \cdot k_2 \cdot e^{b'_{n+m-1} \cdot L \cdot (1+L)} \cdot \|u\| \cdot \eta_{n+m-2} + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\leq \prod_{j=1}^4 k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^4 b'_{n+m-j}} \|d(x_{n+m-4}, p)\| \\
&\quad + \prod_{j=1}^4 k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^3 b'_{n+m-j}} \cdot \|u\| \cdot \eta_{n+m-4} \\
&\quad + \prod_{j=1}^3 k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^2 b'_{n+m-j}} \cdot \|u\| \cdot \eta_{n+m-3} \\
&\quad + k_1 \cdot k_2 \cdot e^{L \cdot (1+L) \cdot b'_{n+m-1}} \cdot \|u\| \cdot \eta_{n+m-2} \\
&\quad + k_1 \cdot \|u\| \cdot \eta_{n+m-1} \\
&\quad \vdots \\
&\leq \prod_{j=1}^m k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^m b'_{n+m-j}} \cdot \|d(x_n, p)\| + \prod_{j=1}^m k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^{m-1} b'_{n+m-j}} \cdot \|u\| \cdot \eta_n \\
&\quad + \prod_{j=1}^{m-1} k_j \cdot e^{L \cdot (1+L) \cdot \sum_{j=1}^{m-2} b'_{n+m-j}} \cdot \|u\| \cdot \eta_{n+1} \\
&\quad \vdots \\
&\quad + \prod_{j=1}^3 k_j \cdot e^{L \cdot (1+L) \cdot (b'_{n+m-1} + b'_{n+m-2})} \cdot \|u\| \cdot \eta_{n+m-3} \\
&\quad + k_1 \cdot k_2 \cdot e^{L \cdot (1+L) \cdot b'_{n+m-1}} \cdot \|u\| \cdot \eta_{n+m-2} \\
&\quad + k_1 \cdot \|u\| \cdot \eta_{n+m-1}
\end{aligned}$$

for each corresponding normal constant $k_i (i = 1 \cdots m)$.

Hence $\|d(x_{n+m}, p)\| \leq K \cdot M_1 \cdot \|d(x_n, p)\| + K \cdot M_1 \cdot \|u\| \cdot \sum_{j=n}^{n+m-1} \eta_j$,

where $K = \max(\prod_{j=1}^m k_j, \prod_{j=1}^{m-1} k_j, \prod_{j=1}^{m-2} k_j, \dots, k_1 \cdot k_2, k_1, 1)$.

Thus we have for $m + n, n > N_1$,

$$\begin{aligned}
 \|d(x_{n+m}, x_n)\| &\leq \|d(x_{n+m}, p_1)\| + \|d(x_n, p_1)\| \\
 &\leq D_1 \cdot M_1 \cdot \|d(x_{N_1}, p_1)\| + D_1 \cdot M_1 \cdot \|u\| \cdot \sum_{k=N_1}^{n+m-1} \eta_k \\
 &\quad + D_2 \cdot M_1 \cdot \|d(x_{N_1}, p_1)\| + D_2 \cdot M_1 \cdot \|u\| \cdot \sum_{k=N_1}^{n-1} \eta_k \\
 &\leq (D_1 + D_2) \cdot M_1 \cdot \frac{\varepsilon}{4M_1} + (D_1 + D_2) \cdot M_1 \cdot \|u\| \cdot \frac{\varepsilon}{4 \cdot M_1 \cdot \|u\|} \\
 &= D \cdot \varepsilon \text{ for } D_1 \text{ and } D_2 \text{ obtained as the same method in which} \\
 &\quad K \text{ is obtained, where } D = \max(D_1, D_2).
 \end{aligned}$$

On the other hand, for $c \in E$ with $0 << c$, there exists a positive number δ such that for $d \in E$ with $\|d\| < \delta$, $c - d \in \text{int}P$. From the fact that $d(x_n, x_m) \rightarrow 0$ (as $n, m \rightarrow \infty$) in E , for such δ , there exists a natural number N such that for all $n, m \geq N$ $\|d(x_n, x_m)\| < \delta$. Thus $c - d(x_n, x_m) \in \text{int}P$, which means that $\{x_n\}$ is a Cauchy sequence in C by Proposition 1.1. Since X is complete and C is closed, $\{x_n\}$ converges to some point p^* in C .

(3) Lastly, we show that $p^* \in F$. Let $\{p_n\}$ be a sequence in F such that $p_n \rightarrow p'$.

Since

$$\begin{aligned}
 d(p', T_i p') &\preceq d(p', p_n) + d(p_n, T_i p') \\
 &= d(p', p_n) + d(T_i p_n, T_i p') \\
 &\preceq d(p', p_n) + L \cdot d(p_n, p') \\
 &\rightarrow 0 \text{ (as } n \rightarrow \infty),
 \end{aligned}$$

$d(p', T_i p') = 0$ for $i \in \mathbb{N}$. Similarly, $d(p', S_i p') = 0$ for $i \in \mathbb{N}$.

Thus $p' \in F$, which means that F is closed. Since $d(p^*, F) = \lim_{n \rightarrow \infty} d(x_n, F) = 0$ we have, $p^* \in F$, which completes the proof.

Remark 2.1. Taking bounded constants $L_i = \sup_{n \geq 0} k_{i_n}$ and $L'_i = \sup_{n \geq 0} k'_{i_n}$ for asymptotically quasi-nonexpansive mappings and asymptotically nonexpansive mappings such that $\lim_{n \rightarrow \infty} k_{i_n} = \lim_{n \rightarrow \infty} k'_{i_n} (i \in \mathbb{N}) = 1$, we obtain the same results in Theorem 2.2

Remark 2.2. If (E, P) is a Banach space, then the convex cone metric spaces need not be complete in Theorem 2.2, we obtain the same results as follows; continued from the fact that $\|d(x_{n+m}, x_n)\| \leq K \cdot \varepsilon$ in the second part (2) of the proof.

Since $d(x_n, x_m) \rightarrow 0$ (as $n, m \rightarrow \infty$) in a Banach space E , there exists a point x in E such that $d(x_n, x) \rightarrow 0$ (as $n \rightarrow \infty$).

On the other hand, for $c \in E$ with $0 << c$, there exists a positive number δ such that for $\|d\| < \delta$, $c - d \in \text{int}P$. From the fact that $d(x_n, x) \rightarrow 0$ (as $n \rightarrow \infty$) in E , for such δ , there exists a natural number N such that $c - d(x_n, x) \in \text{int}P$, which implies that $d(x_n, x) << c$. Since C is closed, $\{x_n\}$ converges to x in C .

Remark 2.3. Restricting a normed linear space (E, P) to a real number system $(\mathbb{R}, [0, \infty])$, we obtain the same results as Theorem 2.2 in complete convex metric spaces.

Remark 2.4. By reducing the convex cone metric space (X, d, W) in Theorem 2.2 to the convex metric space in Theorem 2 in [1], but without changing all assumptions in Theorem 2.2, we obtain Theorem 2 in [1] as a direct corollary for the sequence $\{x_n\}$ defined as

$$\begin{cases} x_{n+1} = W(x_n, T_n^n y_n, u_n; \alpha_n, \beta_n, \gamma_n), \\ y_n = W(x_n, S_n^n y_n, v_n; a_n, b_n, c_n), \end{cases}$$

which is the Ishikawa-type iterative sequence with errors considered in [5].

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