Hysteresis Modeling and Synchronization of a Class of RC-OTA Hysteretic-Jounce-Chaotic Oscillators

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Abstract A class of RC-OTA hysteretic-chaotic oscillators has been previously reported using electronics; therefore, hysteresis is realized by an electronic circuit. To obtain a mathematical model of this RC-OTA chaotic-electronic device, hysteresis modeling turns an important issue. Here, we develop a new mathematical hysteretic model proposing a new jounce-chaotic oscillator. Chaosity test is proved using Poincaré theory. After that, a synchronization scheme is granted to synchronize our new jounce-chaotic oscillator (the transmitter) to a dynamics second-order system (the receiver).

Keywords Chaos, Synchronization, RC-OTA oscillator

1 Introduction

During recent years, many chaotic oscillators have been proposed. Some of them are based on hysteresis feedback ([11], [9]). However, these oscillators have been designed from electronic point of view. Nevertheless, hysteresis modeling is an important issue in mechanical and structural systems ([7]). Some hysteresis models are developed invoking physical laws. Meanwhile, others are heuristic ones. Moreover, [8] reported chaotic behavior in structures with hysteresis, in which hysteresis is governed by the well-known Bouc-Wen model. However, this Bouc-Wen model is not appropriate for the class of RC-OTA chaotic oscillators because it has more parameters than needed. Following this line of *engineering* modeling, we propose a new dynamic-hysteretic model which is able to capture hysteresis behavior for a class of RC-OTA hysteretic-chaotic oscillators designed, for instance, in [9] (where a pioneer study is given in [2]).

In mechanics, a *jerk* function is the time derivative of acceleration which, actually, is a third-order dynamic

system. According to [10], some forms of jerk functions present chaos. And the time derivative of a jerk function might be called *jounce* ([10]), which is a fourth-order dynamic system. Using our dynamic-hysteretic model, and inspired by the RC-OTA architecture, we propose a fourth-order chaotic system named jounce-chaotic oscillator, and whose chaosity test is realized through Poincaré theory.

By the other hand, chaos synchronization has gained an important attention among scientist (see, for instance, [1], [4], and [13], among others). This because some fields of engineering issues, e.g. secure communications, use chaos synchronization ([13], [1], and [3]). So far, many of the synchronization systems developed are granted to achieve chaos synchronization between two identical chaotic systems. The case of two different systems seems not to be completely developed ([13]). We also present a synchronization scheme where our jounce-chaotic system is synchronized with a second order system.

The structure of the paper is as follows. Section two presents a review of the *simple* mathematical model of a class of RC-OTA hysteretic-chaotic oscillators. Our new dynamic-hysteretic model is presented in Section three together with numerical experiments. Our new jounce-chaotic system is commented too. In Section four, our synchronization design is granted. Finally, Section five presents the conclusions.

2 RC-OTA hysteretic chaotic systems

According to [9], a dimensionless dynamic model of a class of RC-OTA hysteretic-chaotic oscillator is given by:

$$\ddot{x} - 2\delta\dot{x} + x = ph(x), \quad (1)$$

where δ and p are the system parameters. The hysteresis function $h(x)$ is shown in Fig. 1. This system presents chaos with $\delta = 0.05$, and $p = 1$ ([9]).

Figure 1. Normalized hysteretic function.

3 Hysteresis modeling and a jounce-chaotic system

Hysteresis behavior is recognized as a system with memory. One way to capture hysteresis is by using a dynamic system. For instance, the new hysteretic system:

$$\dot{z} = \alpha(-z + b \operatorname{sgn}(x + a \operatorname{sgn}(z))), \quad (2)$$

can reproduce the hysteretic behavior shown in Fig. 2, where a and b are the hysteresis curve parameters. The speed transition between b and $-b$ is governed by the positive parameter α ; $\operatorname{sgn}(\cdot)$ is the signum function. For instance, if $a = b = 1$ and $\alpha = 10$, the system (2) is:

$$\dot{z} = 10(-z + \operatorname{sgn}(x + \operatorname{sgn}(z))). \quad (3)$$

Using $x = 10 \sin(t)$ and $z(0) = 0$, the simulation result of system (3) is shown in Fig. 3.

Next, we program a chaotic oscillator equivalent to (1):

$$\begin{aligned} \ddot{x} &= -0.1\dot{x} + x + z \\ \dot{z} &= 10(-z + \operatorname{sgn}(x + \operatorname{sgn}(z))). \end{aligned}$$

Fig. 4 shows the simulation result. The obtained chaotic attractor is the same as that shown in [9, Fig. 6].

Figure 2. Hysteretic behavior.

To extend the previous system into a jounce-chaotic realization, consider the next propose:

Figure 3. Simulation result.

Figure 4. Chaotic attractor with initial conditions $x(0) = \dot{x}(0) = 0$ and $z(0) = 0.1$.

$$\dot{x}_1 = x_2, \quad (4)$$

$$\dot{x}_2 = z_1 + z_2 + 0.1x_2 - x_1, \quad (5)$$

$$\dot{z}_1 = 10(-z_1 + \operatorname{sign}(x_1 - 1 + \operatorname{sign}(z_1)) + 0.5), \quad (6)$$

$$\dot{z}_2 = 10(-z_2 + \operatorname{sign}(x_1 + 1 + \operatorname{sign}(z_2)) - 0.5). \quad (7)$$

Figures 5 and 6 show the jounce-chaotic attractor of the system (4)-(7). Fig.7 shows the Poincaré map for the attractor shown in Fig. 6 using the Poincaré section a plane parallel to the plane z_1 - x_2 and located at $x_1 = 1$. According to [12], this map consists of points filling up a region implying chaos. So, this Poncaré-map experiment is our chaosity test to our jounce-chaotic system. By the other hand, Fig. 8 displays $x_1(t)$ and $x_2(t)$ versus time. With a small change on the initial condition of x_1 from 0 to 0.001 (the remaining initial conditions were the same than the previous numerical experiment), the numerical results are displayed in Fig. 9. We thus have a dynamic system that presents bounded trajectory solutions that is highly sensitive to initial conditions and whose signals are random-like. These are the properties of a chaotic system.

4 Synchronization

Let us introduce the following system:

$$\dot{y}_1 = y_2, \quad (8)$$

$$\dot{y}_2 = z_1 + z_2 + 0.2x_2 - y_1 - 0.1y_2, \quad (9)$$

Figure 5. Jounce-chaotic attractor with initial conditions $x_1(0) = x_2(0) = z_1(0) = 0$ and $z_2(0) = 0.1$.

Figure 6. 3-D jounce-chaotic attractor.

Figure 7. Poincaré Map.

where $z_1 := z_1(t)$, $z_2 := z_2(t)$, and $x_2 := x_2(t)$ arrive from the jounce-chaotic system (4)-(7). At this point, the jounce-chaotic system (4)-(7) represents the transmitter and the system (8)-(9) the receiver. Fig 10 gives a schematic representation of our the synchronization design.

It is said that the receiver is synchronized with the transmitter if $y_1(t)$ converges to $x_1(t)$ and $y_2(t)$ converges to $x_2(t)$ as time goes on (and for any initial conditions

Figure 8. Simulation results: $x_1(t)$ and $x_2(t)$ versus time.

Figure 9. High sensitivity test on the initial conditions: blue line with $x_1(0) = 0$ and red line with $x_1(0) = 0.001$.

Figure 10. Synchronization block diagram.

$y_1(0), x_1(0), y_2(0)$, and $x_2(0)$)¹. This fact can be proved as follows. Consider the signal errors given by

$$\begin{aligned} e_1 &= x_1 - y_1, \\ e_2 &= x_2 - y_2. \end{aligned}$$

Then, after some basic manipulations, we obtain:

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= -e_1 - 0.1e_2. \end{aligned}$$

The above system represents an exponential stable dynamics. Simulation results are shown in Fig. 11.

Remark 1 On synchronization of chaotic systems, it is used to test the synchronization performance by adding a noisy signal to the lines of the channel communication. This noisy signal is a kind of *common noise* because it is induced simultaneously on each communication lines ([5]). But, from the technological point of view, common noisy signals are easily to remove via Instrumentation Amplifiers. For instance, according to ([5], page 85), the common noise induced

¹Note that signals $z_1(t)$ and $z_2(t)$ are bounded for all $t \geq 0$.

Figure 11. Simulation results with initial conditions $x_1(0) = x_2(0) = z_1(0) = 0$, $z_2(0) = 0.1$, $y_1(0) = 3$ and $y_2(0) = -3$.

on the communication lines can be reduced 140dB; i.e., it can be attenuate by a factor of 10^{14} . By the other hand, employing fiber optics, noisy signals are practically unnoticed on our communication system.

Remark 2 The communication scheme shown in Fig. 10 requires three lines of communication. However, it is possible to implement a multiplexing communication technique to drive a single line of communication (Website <http://en.wikipedia.org/wiki/Multiplexing>).

5 Conclusions

A new hysteretic-jounce-chaotic system has been designed along with a synchronization scheme. According to numerical experiments, chaos synchronization between two different systems can be achieved. This fact was theoretically proved too.

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