Interference of Impulses, Scattered and Radiated by Bodies of Spheroidal Form

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Abstract With the help of the Fourier transform and characteristics of the scattering of the stationary (continuous) sound signal are calculated the interference of the impulses, scattered and radiated by bodies of the prolate spheroidal form (ideal and elastic), with the harmonic and the frequency-modulated filling.

Keywords Interference, Spheroidal Body, Elastic Body, Boundary Conditions

1. Introduction

For ideal prolate spheroids, imitating the school, are calculated the interference of scattered impulses, but for elastic spheroidal shells – the interference of radiated impulses (with the help of the dynamic theory of the elasticity, characteristics of the scattering of the stationary sound and the reciprocity theorem). By the scattering of sound by bodies of the spheroidal form had devoted works [1 – 8], by experiments – [9, 10].

2. Interference of Impulses, Scattered by the School

The interference of impulses with the harmonic filling, reflected by the school, are learnt

In [11]. In this paper we will study signals in the form of impulses with the harmonic or frequency – modulated filling. At first we consider the school from three fishes, approximated by three soft prolate spheroids (see Fig. 1), illuminated impulses with the harmonic or frequency – modulated filling. The distance between scatterers are chosen with the help of calculations, fulfilled in [12]. In the process of calculations are found time responses and modulus of spect-rums of scattered impulses of separate scatterers and the summarized reflected impulse. The angle of the illumination θ_0 was taken three values: 30° , 60° , 90° . For the impulse of the illumination $\Psi_i(t)$ with the harmonic filling of the frequency $\omega_0 = 2\pi\nu_0$ his spectrum $S_0(2\pi\nu)$ has the appearance [13]:

$$S_0(2\pi\nu) = \frac{i\nu_0}{\pi(\nu_0^2 - \nu^2)} (-1)^n \sin(\pi n\nu / \nu_0) \quad (1)$$

where T – the period of oscillations with the frequency ν_0 , $T = 1/\nu_0$; n – the number of periods in the impulse; ν – the frequency. The impulse of the illumination $\Psi_i(t)$ and the modulus of his spectrum $S_0(\nu)$ are presented at Fig.2.

Figure 1. The plan of the school from three fishes: a) the side view; b) the view from above.

Figure 2. The impulse of the illumination or the excitation with the harmonic filling $\Psi_i(t)$ (a) and the normalized modulus of his spectrum $|S_0(\nu)|$ (b)

In the frequency – modulated impulse $\Psi_0(t)$ the frequency is changed at the linear dependence:

$$\omega = \omega_0 + at \quad (2)$$

where: $a = 1,626 \cdot 10^6$

The spectrum $S_{01}(2\pi\nu)$ of the frequency – modulated impulse is determined at the formula:

$$S_{01}(2\pi\nu) = \int_{-nT/2}^{+nT/2} e^{-i2\pi\nu t} \cdot \sin[(2\pi\nu_0 + at) \cdot t] dt \quad (3)$$

The impulse of the illumination with the frequency – modulated filling $\Psi_0(t)$ and the modulus his spectrum $S_0(\nu)$ are represented at Fig. 3.

Figure 3. The impulse of the illumination or the excitation with the frequency – modulated filling $\Psi_0(t)$ (a) and the normalized modulus of his spectrum $|S_{01}(\nu)|$ (b).

The spectrum of the scattered signal at the set direction $\theta, \varphi - S_s(2\pi\nu; \theta, \varphi)$ and the spectrum of the radiated signal at the direction $\theta, \varphi - S_q(2\pi\nu; \theta, \varphi)$ were found as the product of the spectrum $S_0(2\pi\nu)$ or $S_{01}(2\pi\nu)$ incidenting (the problem of the diffraction) or exciting (the problem of the radiation) impulses on the frequency response of the scattering $D_s(2\pi\nu; \theta, \varphi)$ or the frequency response of the radiation $D_q(2\pi\nu; \theta, \varphi)$ according. With the help $S_s(2\pi\nu; \theta, \varphi)$ and $S_q(2\pi\nu; \theta, \varphi)$ are found imsges of scattered $[\Psi_s(t)]$ and radiated $[\Psi_q(t)]$ impulses:

$$\Psi_s(t') = \pi^{-1} \operatorname{Re} \int_0^\infty S_s(2\pi\nu; \theta, \varphi) \cdot e^{+i2\pi\nu t'} \cdot d(2\pi\nu) \quad (4)$$

$$\Psi_q(t') = \pi^{-1} \operatorname{Re} \int_0^\infty S_q(2\pi\nu; \theta, \varphi) \cdot e^{+i2\pi\nu t'} \cdot d(2\pi\nu) \quad (5)$$

For the determination of spectrums $S_s(2\pi\nu; \theta, \varphi)$ and $S_q(2\pi\nu; \theta, \varphi)$ are used characteristics of the scattering or the radiation of the sound by spheroidal bodies for the harmonic signal [14]. On the Fig. 4 are represented scattered impulses for physical models of three fishes, finding at distances r_1, r_2 and r_3 , at that $r_2 - r_1 \ll r_1$ and $r_3 - r_1 \ll r_1$ (Fig. 4), the first scatterer in 1,5 times as large of the second scatterer and in 1,2 times as large of the third scatterer. The time t_0 corresponds to the time of the transit of the scattered signal from the first body until the point of the observation, that is $t_0 = r_1 / c$, where c – the velocity of the sound in the liquid. From the interference of third scattered impulses is arisen the signal $\Psi_{s_\Sigma}(t)$ (Fig. 5-a), at Fig.5-b represents the normalized modulus of his spectrum $|S_{s_\Sigma}(\nu)|$, at Fig. 4 and 5 the angle of the illumination θ_0 is equal 90° .

Figure 4. Scattered impulses with frequency – modulated filling for three bodies of the spheroidal form of different sizes; $\theta_0 = 90^\circ$.

Figure 5. The summarized reflected impulse with frequency – modulated filling $\Psi_{S_1}(t)$ (a) and the normalized modulus of his spectrum $|S_{S_1}(\nu)|$ (b); $\theta_0 = 90^\circ$

The same calculation by the angle of the illumination $\theta_0 = 90^\circ$ was fulfilled for impulses with the harmonic filling (see Fig. 6 and 7).

Figure 6. Reflected impulses with the harmonic filling for three bodies of spheroidal form of different sizes, $\theta_0 = 90^\circ$.

Figure 7. The summarized reflected impulse with thr harmonic filling $\Psi_{s_2}(t)$ (a) and the normalized modulus of his spectrum $|S_{s_2}(v)|$ (b).

The comparison of Fig. 5 and 7 demonstrates, what impulses with harmonic filling more stable to the interference, that impulses with the frequency – modulated filling.

3. The Interference of Radiated Impulses

At Fig. 8 is represented the plan of the disposition of two elastic prolate spheroidal shells, exciting in points A_1 and A_2 by impulses with harmonic (Fig. 2) and frequency - modulated (Fig. 3) fillings, the disposition of points A_1 and A_2 corresponds by the spheroidal angle $\theta_0 = 0^\circ$ (the axis – symmetrical problem)

Figure 8. The plan of two spheroidal shell, exciting in points A_1 and A_2 by pulsed signals with the frequency – modulated filling (A_1) and the harmonic filling (A_2); $\theta_0 = 0^\circ$

This problem was decided in [14 – 16], we take advantage of this solution. Let us consider a scatterer in the form of an isotropic spheroidal shell, illuminating along axis of the rotation of the shell (the axis – symmetrical problem). All the potentials, including the plane wave potential Φ_0 , the scattered wave potential Φ_1 , the scalar shell potential Φ_2 , the component A_φ of the vector $\vec{A}$ potential and the potential Φ_3 of the gas filling the shell, can be expanded in spheroidal wave functions [14 – 16]:

$$\Phi_0 = 2 \sum_{n=0}^{\infty} i^{-n} \bar{S}_{0,n}(C, 1) \bar{S}_{0,n}(C, \eta) R_{0,n}^{(1)}(C, \xi); \quad (6)$$

$$\Phi_1 = 2 \sum_{n=0}^{\infty} B_n \bar{S}_{0,n}(C, \eta) R_{0,n}^{(3)}(C, \xi); \quad (7)$$

$$\Phi_2 = 2 \sum_{n=0}^{\infty} [C_n R_{0,n}^{(1)}(C_l, \xi) + D_n R_{0,n}^{(2)}(C_l, \xi)] \bar{S}_{0,n}(C_l, \eta); \quad (8)$$

$$A_\varphi = 4 \sum_{n=1}^{\infty} [F_n R_{1,n}^{(1)}(C_t, \xi) + G_n R_{1,n}^{(2)}(C_t, \xi)] \bar{S}_{1,n}(C_t, \eta); \quad (9)$$

$$\Phi_3 = 2 \sum_{n=0}^{\infty} E_n R_{0,n}^{(1)}(C_1, \xi) \bar{S}_{0,n}(C_1, \eta), \quad (10)$$

where: $\bar{S}_{m,n}(C_l, \eta)$ - the angular spheroidal function; $R_{m,n}^{(1)}(C_l, \xi)$, $R_{m,n}^{(2)}(C_l, \xi)$ and $R_{m,n}^{(3)}(C_l, \xi)$ - radial spheroidal functions of first, second and third genders corresponding;

$C_l = k_l h_0, k_l$ - the wavenumber of the longitudinal elastic wave, h_0 - semi - focus distance; $C_t = k_t h_0, k_t$ - the wavenumber of the transverse elastic wave; $C = k h_0, k$ - the wavenumber of the sound wave in the fluid; $C_1 = k_1 h_0, k_1$ - the wavenumber of the sound wave in the gas filling the shell; $B_n, C_n, D_n, F_n, G_n, E_n$ are unknown expansion coefficients. The expansion coefficients are determined from the physical boundary conditions preset at the two surfaces of the shell (ξ_0 and ξ_1):

- 1) the continuity of the normal displacement component at both of the boundaries, ξ_0 and ξ_1 ;
- 2) the identity between the normal stress in the elastic shell and the sound pressure in the liquid (ξ_0) or in the gas (ξ_1);
- 3) the absence of tangential stress at both of the shell boundaries, ξ_0 and ξ_1 .

The corresponding expressions for the boundary conditions have in the form [14 – 16]:

$$-(h_\xi)^{-1}(\partial / \partial \xi)(\Phi_0 + \Phi_1) = -(h_\xi)^{-1}(\partial \Phi_2 / \partial \xi) + (h_\eta h_\varphi)^{-1}[(\partial / \partial \eta)(h_\varphi A_\varphi)] \text{ by } \xi = \xi_0; \quad (11)$$

$$-(h_\xi)^{-1}(\partial \Phi_3 / \partial \xi) = -(h_\xi)^{-1}(\partial \Phi_2 / \partial \xi) + (h_\eta h_\varphi)^{-1}[(\partial / \partial \eta)(h_\varphi A_\varphi)] \text{ by } \xi = \xi_1; \quad (12)$$

$$\begin{aligned} \Lambda_0 k^2 (\Phi_0 + \Phi_1) = \Lambda_1 k_l^2 \Phi_2 + 2\mu_1 \{ & -(h_\xi h_\eta)^{-1}(\partial h_\xi / \partial \eta)[(h_\eta)^{-1}(\partial \Phi_2 / \partial \eta) + (h_\xi h_\varphi)^{-1} \partial (h_\varphi A_\varphi) / \partial \xi] + \\ & + (h_\xi)^{-1}(\partial / \partial \xi)[-(h_\xi)^{-1}(\partial \Phi_2 / \partial \xi) + (h_\eta h_\varphi)^{-1} \partial (h_\varphi A_\varphi) / \partial \eta] \} \text{ by } \xi = \xi_0; \quad (13) \end{aligned}$$

$$\begin{aligned} \Lambda_2 k_1^2 \Phi_3 = \Lambda_1 k_l^2 \Phi_2 + 2\mu_1 \{ & -(h_\xi h_\eta)^{-1}(\partial h_\xi / \partial \eta)[(h_\eta)^{-1}(\partial \Phi_2 / \partial \eta) + (h_\xi h_\varphi)^{-1} \partial (h_\varphi A_\varphi) / \partial \xi] + \\ & + (h_\xi)^{-1}(\partial / \partial \xi)[-(h_\xi)^{-1}(\partial \Phi_2 / \partial \xi) + (h_\eta h_\varphi)^{-1} \partial (h_\varphi A_\varphi) / \partial \eta] \} \text{ by } \xi = \xi_1; \quad (14) \end{aligned}$$

$$-[(h_\xi h_\eta)^{-1}(\partial h_\eta / \partial \xi) - (h_\xi)^{-1}(\partial / \partial \xi)][(h_\eta)^{-1}(\partial \Phi_2 / \partial \eta) + (h_\xi h_\varphi)^{-1} \partial (h_\varphi A_\varphi) / \partial \xi] + [(h_\xi h_\eta)^{-1}(\partial h_\xi / \partial \eta) -$$

$$-(h_{\eta})^{-1}(\partial/\partial\eta)[-(h_{\xi})^{-1}(\partial\Phi_2/\partial\xi)+(h_{\phi}h_{\eta})^{-1}\partial(h_{\phi}A_{\phi})/\partial\eta]=0 \quad \text{by } \xi=\xi_0 \text{ and } \xi=\xi_1, (15)$$

where $h_{\xi}=h_0(\xi^2-\eta^2)^{1/2}(\xi^2-1)^{-1/2}$; $h_{\eta}=h_0(\xi^2-\eta^2)^{1/2}(1-\eta^2)^{-1/2}$; $h_{\phi}=h_0[(1-\eta^2)(\xi^2-1)]^{1/2}$;

Λ_0 is the bulk coefficient of the liquid; Λ_1 and μ_1 are Lamé constants of the spheroidal shell;

Λ_2 is the bulk compression coefficient of the gas filling the shell.

The substitution of series (6) – (10) in boundary conditions (11) – (15) yields an infinite system of equations for determining the desired coefficients. The infinite system is solved by the truncation method. The number of retained terms of expansions (6) – (10) is the greater, the greater the wave size for the given potential.

At the Fig. 9 are represented responses of shells an exciting impulses with the frequency – modulated filling (a) and the harmonic filling (b), but at Fig. 10 is presented summarized impulse

$\Psi_{q_{\Sigma}}(t)$ (a) and the normalized modulus of his spectrum $|S_{q_{\Sigma}}(\nu)|$ (b).

The presence in the summarized impulse of the impulse with the harmonic filling smoothes over the negative action of the interference at the summarized impulse.

Figure 9. Responses of shells at impulses with the frequency – modulated filling (a) and the harmonic filling (b);

Figure 10. The summarized impulse of the response $\Psi_{q_{\Sigma}}(t)$ in the point of the observation and the normalized modulus of his spectrum $|S_{q_{\Sigma}}(\nu)|$;

$$\theta_0 = 0^\circ$$

The stable character of impulses with harmonic filling can see from the comparison of Fig. 11 and 12 (the frequency –

modulated filling) with Fig. 13 and 14 (the harmonic filling).

The advantage of the application of impulses with the harmonic filling by the interference appears evidently.

Figure 11. Responses of three spheroidal shells at exciting impulses with the frequency – modulated filling

Figure 12. The summarized impulse of the response with the frequency – modulated filling $\Psi_{q_z}(t)$ in the point of the observation and the normalized modulus of his spectrum $|S_{q_z}(\nu)|$; $\theta_0 = 0^\circ$.

Figure 13. Responses of three spheroidal shells at exciting impulses with the harmonic filling

Figure 14. The summarized impulse of the response with the harmonic filling $\Psi_{q_{\Sigma}}(t)$ in the point of the observation and the normalized modulus of his spectrum $|S_{q_{\Sigma}}(\nu)|$; $\theta_0 = 0^\circ$

4. Conclusions

In the paper were calculated scattered and radiated impulses with harmonic and frequency – modulated fillings for ideal and elastic spheroidal bodies. Signals with the harmonic filling appear more stable to the interference. The study of the interference of impulses, scattered and radiated by bodies of the spheroidal form appears by the object of the research in the hydroacoustic.

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