A Unique Common Fixed Point Theorem Using $\psi - \phi$ Condition in a Partial Metric Space Using an ICS Mapping

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Abstract The ICS mapping was introduced by K.P.Chi [On a fixed point theorem for certain class of maps satisfying a contractive condition depended on an another function, Lobachevskii J. math., 30(4), 2009, 289 - 291.] In this paper, we obtain a unique common fixed point theorem in partial metric spaces by using ICS mapping and also introduced supported example to our main theorem.

Keywords Partial Metric, ICS Mapping, Complete Space

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1 Introduction and Preliminaries

The notion of partial metric space was introduced by Matthews [10] as a part of the study of denotational semantics of data flow networks. In fact, it is widely recognized that partial metric spaces play an important role in constructing models in the theory of computation and domain theory in computer science (see e.g. [12, 13, 14, 15, 16, 17, 18, 19, 20]).

Matthews [10, 11], Oltra and Valero [23] and Romaguera [21] and Altun et al. [2] proved some fixed point theorems in partial metric spaces for a single map (see also [1, 6, 7, 8, 9, 3, 4, 22, 17]).

First we recall some basic definitions and lemmas which play crucial role in the theory of partial metric spaces.

Definition 1.1 (See [10, 11]) A partial metric on a nonempty set X is a function $p : X \times X \rightarrow \mathbb{R}^+$ such that for all $x, y, z \in X$:

$$(p_1) \quad x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y),$$

$$(p_2) \quad p(x, x) \leq p(x, y), p(y, y) \leq p(x, y),$$

$$(p_3) \quad p(x, y) = p(y, x),$$

$$(p_4) \quad p(x, y) \leq p(x, z) + p(z, y) - p(z, z).$$

The pair (X, p) is called a partial metric space (PMS).

Clearly $p(x, y) = 0$ implies $x = y$ and $x \neq y$ implies $p(x, y) > 0$.

If p is a partial metric on X , then the function $d_p : X \times X \rightarrow \mathbb{R}^+$ given by $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ is a metric on X .

Example 1.2 (See e.g. [11, 7, 1]) Consider $X = [0, \infty)$ with $p(x, y) = \max\{x, y\}$. Then (X, p) is a partial metric space. It is clear that p is not a (usual) metric. Note that in this case $d_p(x, y) = |x - y|$.

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p -balls $\{B_p(x, \varepsilon), x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

We now state some basic topological notions (such as convergence, completeness, continuity) on partial metric spaces (see e.g. [10, 11, 2, 1, 7, 9].)

Definition 1.3

1. A sequence $\{x_n\}$ in the PMS (X, p) converges to the limit x if and only if $p(x, x) = \lim_{n \rightarrow \infty} p(x, x_n)$.
2. A sequence $\{x_n\}$ in the PMS (X, p) is called a Cauchy sequence if $\lim_{n, m \rightarrow \infty} p(x_n, x_m)$ exists and is finite.
3. A PMS (X, p) is called complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p , to a point $x \in X$ such that $p(x, x) = \lim_{n, m \rightarrow \infty} p(x_n, x_m)$.
4. A mapping $F : X \rightarrow X$ is said to be continuous at $x_0 \in X$ if for every $\epsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x_0, \delta)) \subseteq B_p(Fx_0, \epsilon)$.

We need the following lemmas in PMS([10, 11, 1, 2, 7, 9]).

Lemma 1.4

1. A sequence $\{x_n\}$ is a Cauchy sequence in the PMS (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p) .
2. A PMS (X, p) is complete if and only if the metric space (X, d_p) is complete. Moreover

$$\lim_{n \rightarrow \infty} d_p(x, x_n) = 0 \Leftrightarrow p(x, x) = \lim_{n \rightarrow \infty} p(x, x_n) = \lim_{n, m \rightarrow \infty} p(x_n, x_m) \quad (1.1)$$

Lemma 1.5 Assume $x_n \rightarrow z$ as $n \rightarrow \infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then $\lim_{n \rightarrow \infty} p(x_n, y) = p(z, y)$ for every $y \in X$.

In this paper, we obtain a unique common fixed point theorem for two mappings using ICS mapping in Partial metric spaces. Our result generalizes the recent several known results.

Recently [5] introduced the concept of ICS mapping as follows.

Definition 1.6 [5] Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ is said to be ICS if T is injective, continuous and has the property : for every sequence $\{x_n\}$ in X , if $\{Tx_n\}$ is convergent then $\{x_n\}$ is also convergent.

2 MAIN RESULT

Let Ψ denote the set of all continuous and monotonically increasing functions $\psi : [0, \infty) \rightarrow [0, \infty)$.

Let Φ denote the set of all lower semi continuous functions $\phi : [0, \infty) \rightarrow [0, \infty)$ such that $\phi(t) > 0$ for $t > 0$.

Theorem 2.1 Let (X, p) be a partial metric space and $T : X \rightarrow X$ be an ICS mapping and $F, G : X \rightarrow X$ be satisfying

$$\begin{aligned} \psi(p(TFx, TGy)) &\leq \psi\left(\max\left\{\begin{array}{l} p(Tx, Ty), p(Tx, TFx), p(Ty, Tgy), \\ \frac{1}{2}[p(Tx, TGy) + p(Ty, TFx)] \end{array}\right\}\right) \\ &\quad - \phi\left(\max\left\{\begin{array}{l} p(Tx, Ty), p(Tx, TFx), p(Ty, Tgy), \\ \frac{1}{2}[p(Tx, TGy) + p(Ty, TFx)] \end{array}\right\}\right), \end{aligned}$$

$\forall x, y \in X$, where $\psi \in \Psi$ and $\phi \in \Phi$. Then F and G have a unique common fixed point in X .

Let $x_0 \in X$. Define $x_{2n+1} = Fx_{2n}, x_{2n+2} = Gx_{2n+1}$, $n = 0, 1, 2, \dots$ and $y_n = Tx_n$, $n = 0, 1, 2, \dots$

Case(a): Suppose $y_{2n+1} = y_{2n}$ for some n .

Then $Tx_{2n+1} = Tx_{2n}$.

Since T is injective, we have $x_{2n+1} = x_{2n} \Rightarrow Fx_{2n} = x_{2n}$.

Thus $F\alpha = \alpha$, where $\alpha = x_{2n}$.
Suppose $T\alpha \neq TG\alpha$. Consider

$$\begin{aligned}\psi(p(T\alpha, TG\alpha)) &= \psi(p(TF\alpha, TG\alpha)) \\ &\leq \psi\left(\max\left\{p(T\alpha, T\alpha), p(T\alpha, TF\alpha), p(T\alpha, TG\alpha), \frac{1}{2}[p(T\alpha, TG\alpha) + p(T\alpha, TF\alpha)]\right\}\right) \\ &\quad - \phi\left(\max\left\{p(T\alpha, T\alpha), p(T\alpha, TF\alpha), p(T\alpha, TG\alpha), \frac{1}{2}[p(T\alpha, TG\alpha) + p(T\alpha, TF\alpha)]\right\}\right) \\ &= \psi(p(T\alpha, TG\alpha)) - \phi(p(T\alpha, TG\alpha)), \text{ from } (p_2) \\ &< \psi(p(T\alpha, TG\alpha)),\end{aligned}$$

a contradiction.

Hence $T\alpha = TG\alpha$. Since T is injective, we have $\alpha = G\alpha$.

Thus α is a common fixed point of F and G .

If β is another common fixed point of F and G , then $T\alpha \neq T\beta$.

$$\begin{aligned}\psi(p(T\alpha, T\beta)) &= \psi(p(TF\alpha, TG\beta)) \\ &\leq \psi\left(\max\left\{p(T\alpha, T\beta), p(T\alpha, TF\alpha), p(T\beta, TG\beta), \frac{1}{2}[p(T\alpha, TG\beta) + p(T\beta, TF\alpha)]\right\}\right) \\ &\quad - \phi\left(\max\left\{p(T\alpha, T\beta), p(T\alpha, TF\alpha), p(T\beta, TG\beta), \frac{1}{2}[p(T\alpha, TG\beta) + p(T\beta, TF\alpha)]\right\}\right) \\ &= \psi\left(\max\left\{p(T\alpha, T\beta), p(T\alpha, T\alpha), p(T\beta, T\beta), \frac{1}{2}[p(T\alpha, T\beta) + p(T\beta, T\alpha)]\right\}\right) \\ &\quad - \phi\left(\max\left\{p(T\alpha, T\beta), p(T\alpha, T\alpha), p(T\beta, T\beta), \frac{1}{2}[p(T\alpha, T\beta) + p(T\beta, T\alpha)]\right\}\right) \\ &= \psi(p(T\alpha, T\beta)) - \phi(p(T\alpha, T\beta)), \text{ from } (p_2) \\ &< \psi(p(T\alpha, TG\alpha)),\end{aligned}$$

a contradiction.

Thus α is the unique common fixed point of F and G .

Case(b) : Assume that $y_n \neq y_{n+1}$ for all n .

Denote $p_n = p(y_n, y_{n+1})$.

$$\begin{aligned}\psi(p_{2n}) &= \psi(p(y_{2n}, y_{2n+1})) \\ &= \psi(p(Tx_{2n+1}, Tx_{2n})) \\ &= \psi(p(TFx_{2n}, TGx_{2n-1})) \\ &\leq \psi\left(\max\left\{p(y_{2n}, y_{2n-1}), p(y_{2n}, y_{2n+1}), p(y_{2n-1}, y_{2n}), \frac{1}{2}[p(y_{2n}, y_{2n}) + p(y_{2n-1}, y_{2n+1})]\right\}\right) \\ &\quad - \phi\left(\max\left\{p(y_{2n}, y_{2n-1}), p(y_{2n}, y_{2n+1}), p(y_{2n-1}, y_{2n}), \frac{1}{2}[p(y_{2n}, y_{2n}) + p(y_{2n-1}, y_{2n+1})]\right\}\right) \\ &= \psi(\max p_{2n-1}, p_{2n}) - \phi(\max p_{2n-1}, p_{2n}), \text{ from } (p_4)\end{aligned}$$

If p_{2n} is maximum, then $\psi(p_{2n}) \leq \psi(p_{2n}) - \phi(p_{2n}) < \psi(p_{2n})$, a contradiction.

Hence

$$\begin{aligned}\psi(p_{2n}) &\leq \psi(p_{2n-1}) - \phi(p_{2n-1}) \\ &< \psi(p_{2n-1}).\end{aligned}\tag{2.1}$$

$p_{2n} < p_{2n-1}$, since ψ is monotonically non - decreasing.

Similarly, we can show that $p_{2n+1} < p_{2n}$. Thus $\{p_n\}$ is monotonically decreasing sequence of non negative real numbers and hence $\{p_n\}$ converges to some $r \geq 0$. Suppose $r > 0$.

Letting $n \rightarrow \infty$ in (2.1), we get

$$\psi(r) \leq \psi(r) - \phi(r) < \psi(r), \text{ since } \phi(t) > 0 \text{ for } t > 0.$$

It is a contradiction . Hence $r = 0$.

Thus

$$\lim_{n \rightarrow \infty} p(y_n, y_{n+1}) = 0.\tag{2.2}$$

Hence from (p_2) , we get

$$\lim_{n \rightarrow \infty} p(y_n, y_n) = 0.\tag{2.3}$$

From the definition of d_p , using (2.2) and (2.3), we get

$$\lim_{n \rightarrow \infty} d_p(y_n, y_{n+1}) = 0. \quad (2.4)$$

Now we prove that $\{y_n\}$ is a Cauchy sequence in (X, d_p) .

On contrary suppose that $\{y_{2n}\}$ is not Cauchy.

Then there exist an $\epsilon > 0$ and monotone increasing sequences of natural numbers $\{2m_k\}$ and $\{2n_k\}$ such that $n_k > m_k$,

$$d_p(y_{2m_k}, y_{2n_k}) \geq \epsilon \quad (2.5)$$

and

$$d_p(y_{2m_k}, y_{2n_k-2}) < \epsilon. \quad (2.6)$$

From (2.5) and (2.6), we obtain

$$\begin{aligned} \epsilon &\leq d_p(y_{2m_k}, y_{2n_k}) \\ &\leq d_p(y_{2m_k}, y_{2n_k-2}) + d_p(y_{2n_k-2}, y_{2n_k-1}) + d_p(y_{2n_k-1}, y_{2n_k}) \\ &< \epsilon + d_p(y_{2n_k-2}, y_{2n_k-1}) + d_p(y_{2n_k-1}, y_{2n_k}). \end{aligned}$$

Letting $k \rightarrow \infty$ and then using (2.4), we get

$$\lim_{k \rightarrow \infty} d_p(y_{2m_k}, y_{2n_k}) = \epsilon. \quad (2.7)$$

Hence from definition of d_p and (2.3), we have

$$\lim_{k \rightarrow \infty} p(y_{2m_k}, y_{2n_k}) = \frac{\epsilon}{2}. \quad (2.8)$$

Letting $k \rightarrow \infty$ and then using (2.7) and (2.4) in

$$|d_p(y_{2n_k+1}, y_{2m_k}) - d_p(y_{2n_k}, y_{2m_k})| \leq d_p(y_{2n_k+1}, y_{2n_k})$$

we obtain

$$\lim_{k \rightarrow \infty} d_p(y_{2n_k+1}, y_{2m_k}) = \epsilon. \quad (2.9)$$

Hence, we have

$$\lim_{k \rightarrow \infty} p(y_{2n_k+1}, y_{2m_k}) = \frac{\epsilon}{2}. \quad (2.10)$$

Letting $k \rightarrow \infty$ and then using (2.7) and (2.4) in

$$|d_p(y_{2n_k}, y_{2m_k-1}) - d_p(y_{2n_k}, y_{2m_k})| \leq d_p(y_{2m_k-1}, y_{2m_k}),$$

we get

$$\lim_{k \rightarrow \infty} d_p(y_{2n_k}, y_{2m_k-1}) = \epsilon. \quad (2.11)$$

Hence, we have

$$\lim_{k \rightarrow \infty} p(y_{2n_k}, y_{2m_k-1}) = \frac{\epsilon}{2}. \quad (2.12)$$

Letting $k \rightarrow \infty$ and then using (2.11) and (2.4) in

$$|d_p(y_{2m_k-1}, y_{2n_k+1}) - d_p(y_{2m_k-1}, y_{2n_k})| \leq d_p(y_{2n_k+1}, y_{2n_k})$$

we obtain

$$\lim_{k \rightarrow \infty} d_p(y_{2m_k-1}, y_{2n_k+1}) = \epsilon. \quad (2.13)$$

Hence, we get

$$\lim_{k \rightarrow \infty} p(y_{2m_k-1}, y_{2n_k+1}) = \frac{\epsilon}{2}. \quad (2.14)$$

Now,

$$\begin{aligned} \psi(p(y_{2n_k+1}, y_{2m_k})) &= \psi(p(TFx_{2n_k}, TGx_{2m_k-1})) \\ &\leq \psi \left(\max \left\{ \begin{array}{c} p(y_{2n_k}, y_{2m_k-1}), p(y_{2n_k}, y_{2n_k+1}), \\ p(y_{2m_k-1}, y_{2m_k}), \\ \frac{1}{2}[p(y_{2n_k}, y_{2m_k}) + p(y_{2m_k-1}, y_{2n_k+1})] \end{array} \right\} \right) \\ &\quad - \phi \left(\max \left\{ \begin{array}{c} p(y_{2n_k}, y_{2m_k-1}), p(y_{2n_k}, y_{2n_k+1}), \\ p(y_{2m_k-1}, y_{2m_k}), \\ \frac{1}{2}[p(y_{2n_k}, y_{2m_k}) + p(y_{2m_k-1}, y_{2n_k+1})] \end{array} \right\} \right). \end{aligned}$$

Letting $k \rightarrow \infty$ and then using (2.10), (2.12), (2.2), (2.14) and (2.8), we get

$$\begin{aligned}\psi\left(\frac{\epsilon}{2}\right) &\leq \psi\left(\max\left\{\frac{\epsilon}{2}, 0, 0, \frac{1}{2}\left[\frac{\epsilon}{2} + \frac{\epsilon}{2}\right]\right\}\right) - \phi\left(\max\left\{\frac{\epsilon}{2}, 0, 0, \frac{1}{2}\left[\frac{\epsilon}{2} + \frac{\epsilon}{2}\right]\right\}\right) \\ &= \psi\left(\frac{\epsilon}{2}\right) - \phi\left(\frac{\epsilon}{2}\right) \\ &< \psi\left(\frac{\epsilon}{2}\right),\end{aligned}$$

a contradiction. Hence $\{y_{2n}\}$ is Cauchy.

Letting $n, m \rightarrow \infty$ in

$$|d_p(y_{2n+1}, y_{2m+1}) - d_p(y_{2n}, y_{2m})| \leq d_p(y_{2n+1}, y_{2n}) + d_p(y_{2m}, y_{2m+1})$$

we get $\lim_{n, m \rightarrow \infty} d_p(y_{2n+1}, y_{2m+1}) = 0$.

Hence $\{y_{2n+1}\}$ is Cauchy.

Thus $\{y_n\}$ is a Cauchy sequence in (X, d_p) .

Hence, we have $\lim_{n, m \rightarrow \infty} d_p(y_n, y_m) = 0$.

Now, from the definition of d_p and from (2.3), we obtain

$$\lim_{n, m \rightarrow \infty} p(y_n, y_m) = 0. \quad (2.15)$$

Since X is complete and $\{y_n\}$ is a Cauchy sequence in complete metric space (X, d_p) .

Thus

$$\lim_{n \rightarrow \infty} d_p(y_n, Tz) = 0 \quad \text{for some } Tz \in X$$

Also T is an ICS mapping and $\{y_n\} = \{Tx_n\}$ is convergent, it follows that $\{x_n\}$ is convergent to some $z \in X$. i.e. $\lim_{n \rightarrow \infty} p(x_n, z) = p(z, z)$.

Since T is continuous, from above we have $\lim_{n \rightarrow \infty} p(Tx_n, Tz) = p(Tz, Tz)$.

By Lemma 1.4(2), we have that

$$p(Tz, Tz) = \lim_{n \rightarrow \infty} p(Tx_n, Tz) = \lim_{n \rightarrow \infty} p(y_n, Tz) = \lim_{n, m \rightarrow \infty} p(y_n, y_m). \quad (2.16)$$

From (2.15) and (2.16), we have

$$p(Tz, Tz) = 0. \quad (2.17)$$

Suppose $Tz \neq TFz$.

Consider

$$\begin{aligned}\psi(p(TFz, y_{2n+2})) &= \psi(p(TFz, Tx_{2n+2})) \\ &= \psi(p(TFz, TGx_{2n+1})) \\ &\leq \psi \max \left(\left\{ p(Tz, Tx_{2n+1}), p(Tz, TFz), p(Tx_{2n+1}, Tx_{2n+2}), \right. \right. \\ &\quad \left. \left. \frac{1}{2}[p(Tz, Tx_{2n+2}) + p(Tx_{2n+1}, TFz)] \right\} \right) \\ &\quad - \phi \max \left(\left\{ p(Tz, Tx_{2n+1}), p(Tz, TFz), p(Tx_{2n+1}, Tx_{2n+2}), \right. \right. \\ &\quad \left. \left. \frac{1}{2}[p(Tz, Tx_{2n+2}) + p(Tx_{2n+1}, TFz)] \right\} \right) \\ &\leq \psi \max \left(\left\{ p(Tz, y_{2n+1}), p(Tz, TFz), p(y_{2n+1}, y_{2n+2}), \right. \right. \\ &\quad \left. \left. \frac{1}{2}[p(Tz, y_{2n+2}) + p(y_{2n+1}, TFz)] \right\} \right) \\ &\quad - \phi \max \left(\left\{ p(Tz, y_{2n+1}), p(Tz, TFz), p(y_{2n+1}, y_{2n+2}), \right. \right. \\ &\quad \left. \left. \frac{1}{2}[p(Tz, y_{2n+2}) + p(y_{2n+1}, TFz)] \right\} \right)\end{aligned}$$

Letting $n \rightarrow \infty$ and using Lemma 1.5 and (2.17), we get

$$\begin{aligned}\psi(p(TFz, Tz)) &\leq \psi \max \left(\left\{ p(Tz, Tz), p(Tz, TFz), 0, \frac{1}{2}[p(Tz, Tz) + p(Tz, TFz)] \right\} \right) \\ &\quad - \phi \max \left(\left\{ p(Tz, Tz), p(Tz, TFz), 0, \frac{1}{2}[p(Tz, Tz) + p(Tz, TFz)] \right\} \right) \\ &= \psi(p(Tz, TFz)) - \phi(p(Tz, TFz)) \\ &< \psi(p(Tz, TFz)), \text{ since } \phi(t) > 0 \text{ for } t > 0.\end{aligned}$$

It is a contradiction. Hence $TFz = Tz$.

Since T is injective, we have $Fz = z$.

As in case(a), z is the common fixed point of F and G .

Example 2.2 Let $X = [0, 1]$ and $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a complete partial metric space. Let $T : X \rightarrow X$ and $F, G : X \rightarrow X$ defined by $T(x) = \frac{x}{2}$, $F(x) = \frac{x}{2x+4}$ and $G(x) = \frac{x^2}{4x+2}$. Then it is clear that T is an ICS mapping. Define $\psi \in \Psi$, $\phi \in \Phi$ by $\psi(t) = t$ and $\phi(t) = \frac{t}{2}$.

Also

$$\begin{aligned}
 \psi(p(TFx, TGy)) &= \max\left\{\frac{Fx}{2}, \frac{Gy}{2}\right\} \\
 &= \frac{1}{2} \max\left\{\frac{x}{2x+4}, \frac{y^2}{4y+2}\right\} \\
 &= \frac{1}{4} \max\left\{\frac{x}{x+2}, \frac{y^2}{y+\frac{1}{2}}\right\} \\
 &\leq \frac{1}{4} \max\{x, y\} \\
 &= \frac{1}{2} \max\left\{\frac{x}{2}, \frac{y}{2}\right\} \\
 &= \frac{1}{2} p(Tx, Ty) \\
 &\leq \frac{1}{2} \max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TGy), \frac{1}{2} [p(Tx, TGy) + p(Ty, TFx)]\right\} \\
 &= \psi\left(\max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TGy), \frac{1}{2} [p(Tx, TGy) + p(Ty, TFx)]\right\}\right) \\
 &\quad - \phi\left(\max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TGy), \frac{1}{2} [p(Tx, TGy) + p(Ty, TFx)]\right\}\right)
 \end{aligned}$$

Clearly 0 is unique common fixed point of F and G .

Corollary 2.3 Let (X, p) be complete partial metric space and $T : X \rightarrow X$ be an ICS mapping and $F, G : X \rightarrow X$ be satisfying

$$p(TFx, TGy) \leq \varphi\left(\max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TGy), \frac{1}{2} [p(Tx, TGy) + p(Ty, TFx)]\right\}\right),$$

$\forall x, y \in X$, where $\varphi : [0, \infty) \rightarrow [0, \infty)$ is continous function with $\varphi(t) < t$ for $t > 0$.

Then F and G have a unique common fixed point in X .

It follows from Theorem 2.1 if we put $\psi(t) = t$ and $\phi(t) = t - \varphi(t)$ in Theorem 2.1.

If we take $F = G$ in Corollary 2.3, we get

Corollary 2.4 Let (X, p) be complete partial metric space and $T : X \rightarrow X$ be an ICS mapping and $F, G : X \rightarrow X$ be satisfying

$$p(TFx, TFy) \leq \varphi\left(\max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TFy), \frac{1}{2} [p(Tx, TFy) + p(Ty, TFx)]\right\}\right),$$

$\forall x, y \in X$, where $\varphi : [0, \infty) \rightarrow [0, \infty)$ is continous function with $\varphi(t) < t$ for $t > 0$.

Then F has a unique fixed point in X .

Example 2.5 Let $X = [0, 1]$ and $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a complete partial metric space. Let $T : X \rightarrow X$ ddefined by $T(x) = x$, it is clearly T is an ICS mapping and $F : X \rightarrow X$ by $F(x) = \frac{x}{2x+3}$ and $\varphi(t) = \frac{t}{2}$.

Also

$$\begin{aligned}
 p(TFx, TFy) &= \max\left\{\frac{x}{2x+3}, \frac{y}{2y+3}\right\} \\
 &= \frac{1}{2} \max\left\{\frac{x}{x+\frac{3}{2}}, \frac{y}{y+\frac{3}{2}}\right\} \\
 &\leq \frac{1}{2} \max\{x, y\} \\
 &= \frac{1}{2} p(Tx, Ty) \\
 &\leq \frac{1}{2} \max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TFy), \frac{1}{2} [p(Tx, TFy) + p(Ty, TFx)]\right\} \\
 &= \varphi\left(\max\left\{p(Tx, Ty), p(Tx, TFx), p(Ty, TFy), \frac{1}{2} [p(Tx, TFy) + p(Ty, TFx)]\right\}\right)
 \end{aligned}$$

Clearly 0 is unique fixed point of F .

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