Universal Journal of Engineering Science 1(1): 5-16, 2013 http://www.hrpub.org
DOI: 10.13189/ujes.2013.010102

Simplified Biomechanics for a Possible Explanation of the Ancient Greek Long Jump Using Halteres

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Abstract In this paper closed form analytical expressions were derived in order to simulate the possible action of "halteres" used in the ancient Greek long jump. For the sake of simplicity, elementary theory of rigid body dynamics is used, which however is capable of simulating the motion of a hypothetical rigid jumper for whom the Cartesian components of the initial velocity at the take-off and the angular velocity of rotating arms are prescribed. Particular attention is paid on the initial position and the direction of arms' rotation as well as on the role of the amount of masses due to the "halteres". It was found that if at the take-off the upper limbs are upwards, also rotate forwards, whereas at the landing they are almost downwards, the length of the jump increases as the weight of the halters.

Keywords Biomechanics, Greek Long Jump, Halteres, Inertial Propulsion

1. Introduction

It is well known that the most important factor for the distance travelled by an object is its velocity at take-off, both the speed and the angle. The greater the speed at take-off, the longer the trajectory of the center of mass will be. The aforementioned rule is also applicable to the long jump with run-up, despite the fact that the jumper is not a material projectile but a flexible mechanism. Nevertheless, the theory is fully applicable for the center of mass of the jumper. The world record is 8.95 m (wind 0.3 m/s) due to Mike Powell in 1981 (Tokyo).

On the other hand, it is well known that dumb-bells (halteres) were used during the pentathlon of the Delphic games [1,2]. Several vase-paintings show jumpers holding weights (halteres) in each hand. According to an existing epigram, Phayllos jumped 55 feet (16.28 m) while Chronis of Sparta jumped 52 feet (16.66 m) at Olympia during the games of 664 B.C. Halteres were probably used in both the usual and the standing long jump, but the pattern of their motion and their efficiency has been a matter of debate and research [1-10].

This paper tests the hypothesis that in the Greek long jump the hands were on "uplift" at the moment of the take-off, and probably performed more than one rotation forwards (clockwise). The latter comes from previous experience gained from studies on a vertically jumping "antigravity" (inertial propulsion) mechanism based on rotating masses [11-13]. In the latter studies it had been found that the maximum jump is produced when the take-off velocities of the rotating masses were parallel to the object's motion. In this study the validity of the previous finding will be investigated for the long jump with run-up. In contrast, standing long jumps are highly depended on the change of the geometry of jumper's body and will be studied in a forthcoming paper.

2. Materials and Methods

2.1. Assumptions

The proposed method consists of an analytical mechanical model that is based on the following assumptions:
1) The shape of the jumper's body (except of the upper limbs: arms, forearms and hands) has a mass of value M and operates as a non-rotating rigid object during the entire motion, from the take-off point until the landing (touch-down) point.
2) The mass of each halter (weight) is embodied to the lumped mass of the corresponding upper limbs thus leading to an eccentric mass of magnitude m, which rotates at a radius r (eccentricity).
3) The upper limbs are considered to be rigid and rotating at a constant angular velocity ω. The value of r is considered to be constant.
4) At the take-off point the upper limbs are upwards and vertical (preferably at θ_0 = 90 degrees, see Figure 1). Therefore, the vector of relative velocity (ωr) of the halteres with respect to the jumper's body is very near to the horizontal direction.
5) At landing, the upper limbs form an angle, $\Delta\theta$, with the

initial position as shown on the right of Figure 1. The optimum position, $\Delta\theta$, is under investigation.

6) The take-off and landing points are on the same horizontal level.

Based on the above assumptions, it will be clearly shown that the use of halteres always leads to a higher jump length than what is obtained without them, provided the rotation is forwards (clockwise) and the hands are sufficiently in uplift.

2.2. The Mechanical Model

Let us consider that the initial (take-off) velocity of the mass M (jumper's body excluding the upper limps) is a vector $\vec{V}_0$ of magnitude V_0, which forms an angle ϕ_0 with the horizontal line (Figure 1). Therefore, the components of $\vec{V}_0$ will be $u_0 = V_0 \cos\phi_0$ and $v_0 = V_0 \sin\phi_0$ in the horizontal (x) and the vertical (y) directions, respectively. In addition, the lumped mass (m) has an absolute velocity equal to $\left(u_0 + \omega r \sin\theta_0\right)$, provided the arms are oriented at an angle θ_0 as shown on the left of Figure 1. After the take-off and before landing, the jumper's body has a uniform horizontal velocity component $u = \dot{x}_M(t)$ [the dot means the first derivative of the position x_M of the centre of mass with respect to the time], while the velocity of each lumped mass is given by the sum of the moving frame u plus the horizontal component of the relative velocity; therefore its total value will be $\left(u_0 + \omega r \sin\left(\theta_0 - \omega t\right)\right)$.

While in usual analysis we consider that the horizontal velocity component has a constant value v_0, see for example [2, p.224], the rotating masses involved in this work spoil this tradition. Therefore, the variation of the horizontal velocity component is described by the conservation of linear momentum in the x-direction:

$$\begin{aligned} &Mu + 2m\left[u + \omega r \sin\left(\theta_0 - \omega t\right)\right] \\ &= Mu_0 + 2m\left(u_0 + \omega r \sin\theta_0\right), \end{aligned} \tag{1}$$

whence the horizontal velocity u is given by:

$$u = u_0 + \underbrace{\frac{2m}{\left(M+2m\right)}\omega r\left[\sin\theta_0 - \sin\left(\theta_0 - \omega t\right)\right]}_{\text{due to the rotational mass}} \tag{2}$$

Considering that $u = dx_M/dt \equiv \dot{x}_M$ and then integrating (2) in time, the abscissa x_M of the body is given by:

$$\begin{aligned} &x_M(t) = u_0 t + \\ &\underbrace{\frac{2mr}{\left(M+2m\right)}\left\{\left(\omega\sin\theta_0\right)t - \left[\cos\left(\theta_0 - \omega t\right) - \cos\theta_0\right]\right\}}_{\text{due to rotational mass}} \end{aligned} \tag{3}$$

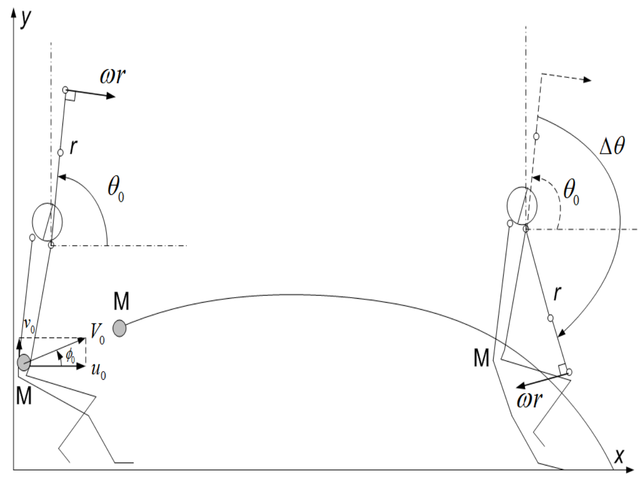

Figure 1. At take-off the jumper is assumed to have the arms at the polar position θ_0 while the centre of mass M has a given initial velocity of components u0 and v0. At landing the arms form an angle Δθ with respect to their initial position.

Then, we apply second Newton's second law in the vertical y-direction (p = vertical component of the total linear momentum):

$$\sum F_y = \frac{dp}{dt} = -\left(M+2m\right)g \tag{4}$$

Integrating (4) in the interval [0,t] one obtains (note: $v = dy_M/dt \equiv \dot{y}_M$):

$$\begin{aligned} &M\left(v - v_0\right) + \\ &2m\left\{\left[v - \omega r\cos\left(\theta_0 - \omega t\right)\right] - \left(v_0 - \omega r\cos\theta_0\right)\right\}, \\ &= -\left(M+2m\right)gt \end{aligned} \tag{5}$$

whence the vertical velocity of the body is given by:

$$v(t) = \left(v_0 - gt\right) + \underbrace{\frac{2m\omega r}{\left(M+2m\right)}\cos\left(\theta_0 - \omega t\right)}_{\text{due to rotational mass}}, \tag{6}$$

and the ordinate of the center of mass of the body M (excluding the arms) is given by:

$$y_M(t) = \left(v_0 t - \frac{1}{2} g t^2\right) + \underbrace{\frac{2mr}{(M+2m)}\left[\sin\theta_0 - \sin(\theta_0 - \omega t)\right]}_{\text{due to rotational mass}} \quad (7)$$

Obviously, in case of non-rotating masses ($\omega = 0$), (3) and (7) degenerate to the well-known parabolic trajectory of jumper's centre of mass. In the latter case, the duration T of the jump is given by:

$$T = \frac{2v_0}{g}, \quad (8)$$

while the length L of the jump is given by:

$$L = u_0 T = \frac{2u_0 v_0}{g}, \quad (9)$$

In case of a rigid object with a given magnitude of the initial velocity, $V_0 = \left(u_0^2 + v_0^2\right)^{1/2}$, the maximum possible length of the shoot is achieved when the inclination angle, ϕ_0, becomes equal to 45 degrees ($u_0 = v_0 = V_0/\sqrt{2}$). Nevertheless, elite jumpers usually leave the ground at an angle of 20 degrees or less.

2.3. Numerical Implementation

We consider a typical athlete's body of weight M = 70kg (not including the arms). The compound tissue "arm, forearm and hands" is considered to have a mass equal to m_a = 3.216 kg while its center of mass is taken r_a = 0.3198 m from the shoulder joint (CM-acromion). The entire arm length was taken equal to L_a = 0.7745 m (see [14, p.38]). Following [2], for the initial velocities we have chosen u_0 = 11 m/s and v_0 = 3 m/s.

Concerning the halteres, the numerical simulation based on the above equations – (3), (4) and (7) – deals with halteres each of them having a weight m_h that varies between 0 and 9 kg. In case that no halteres exist, we consider that m = 3.216 kg (minimum value). In general, we consider that the total rotating mass per arm is m = 3.216 + m_h (kg). Based on the aforementioned parameters, and assuming that the halter does not essentially extend the arm length, i.e. $r_h \cong L_a$, the active eccentricity r of the rotating lumped mass in the model is calculated by the formula:

$$r = \frac{m_a r_a + m_h r_h}{m_a + m_h}, \quad (10)$$

whereas its dependence is shown in Table 1.

Table 1. Rotating lumped mass (m) and eccentricity (r) involved in the analytical mechanical model, for several weights m_k of each halter

Real weight of each halter, m_h (kg)	Corresponding lumped parameters (m, r) of the mechanical model	
	m (kg)	r (m)
0	3.22	0.32
1	4.22	0.43
2	5.22	0.49
3	6.22	0.54
4	7.22	0.57
5	8.22	0.60
6	9.22	0.62
7	10.22	0.63
8	11.22	0.64
9	12.22	0.65

2.4. Increase of Jump Length

Demanding the level of the center of mass of the body M become zero (y_M = 0), (7) is a transcendental equation in t to be solved. When the latter is numerically solved, it produces the desired elapsed time T^* between take-off and landing with $(T^* > T)$, and afterwards, (3) gives the corrected length of jump, L^*, with $L^* > L$. Therefore, with respect to (9), the entire increase in the jump length is given by a sum of two terms as follows:

$$\Delta L = u_0\left(T^* - T\right) + \frac{2mr}{(M+2m)} \cdot \left\{(\omega \sin\theta_0)T^* - \left[\cos\left(\theta_0 - \omega T^*\right) - \cos\theta_0\right]\right\} \quad (11)$$

The first term in (11), $u_0\left(T^* - T\right)$, is due to the increased duration of the jump in conjunction with the invariable horizontal component of the body velocity (no external force is applied towards the x-direction), while the second term is clearly due to the rotational (eccentric) mass, m.

The numerical solution of the nonlinear (11) in T^* was derived using the fzero function of MATLAB® software. Then the jump length L^* was determined through (3).

The results will be presented mostly in terms of the reference angular velocity $\omega_{ref} = \pi/T$, which is defined as the constant angular velocity of the arms when they rotate by $\Delta\theta$ = π radians (180 degrees) during the theoretical jumping period T = 0.6116 seconds (the latter is the duration of jump as calculated using (8) and refers to absence of rotation).

3. Results

3.1. A possible explanation of Phayllos (16.28 m) and Chronis (16.66 m) Renowned Jumps

Figure 2 shows the trajectories for several angular velocities of the rotating arms. It can be noticed that the higher the weight of each halter is, the most posterior and the longest the jump is. The ideal parabolic trajectory (in blue) is obtained setting $m = 0$ and $r = 0$ in (3) and (7). Again, the reference angular velocity is given by $\omega_{ref} = \pi/T$, where according to (8) is T=0.6116 s; the latter gives $\omega_{ref} = 5.14\ \mathrm{s}^{-1} \cong 49.05$ rounds per minute.

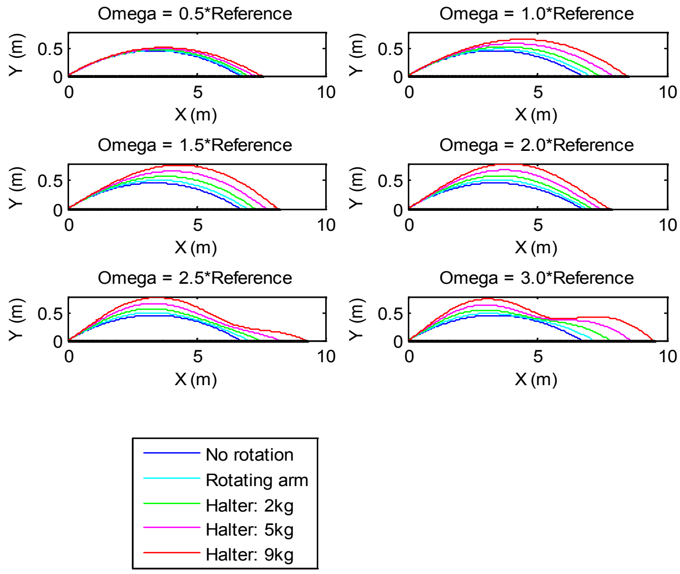

Figure 2. Simulated trajectories of the center of mass based on several angular velocities expressed in terms of the reference value ($\omega_{ref} = 5.14\ \mathrm{s}^{-1}$): ω = [0.5, 1.0, 1.5, 2.0, 2.5, and 3.0] × 5.14 s^{-1}.

Figure 3, Figure 4, Figure 5 and Figure 6 are similar with the previous one but they refer to graphs of progressively higher angular velocities. Again, the higher the weight of each halter is, the most posterior and the longest the jump is.

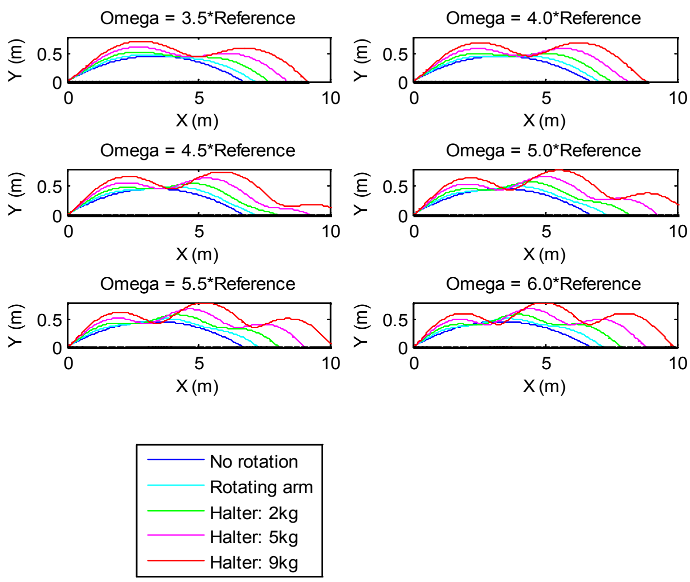

Figure 3. Simulated trajectories of the center of mass based on several angular velocities expressed in terms of the reference value ($\omega_{ref} = 5.14\ \mathrm{s}^{-1}$): ω = [3.5, 4.0, 4.5, 5.0, 5.5, and 6.0] × 5.14 s^{-1}

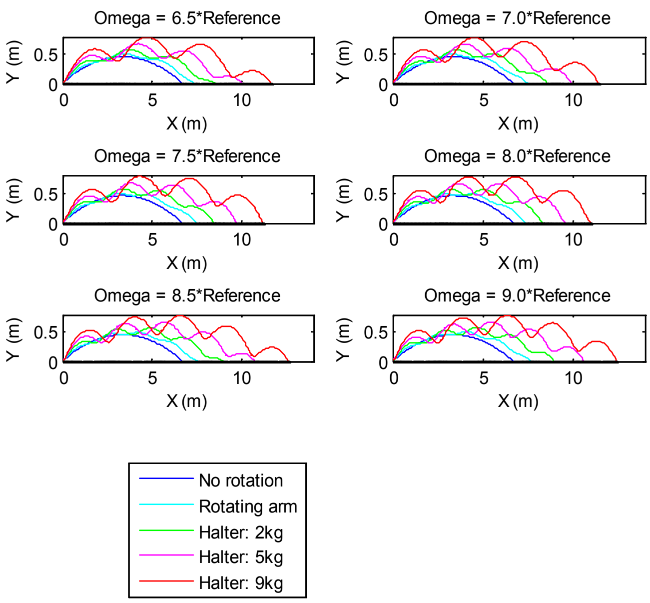

Figure 4. Simulated trajectories of the center of mass based on several angular velocities expressed in terms of the reference value ($\omega_{ref} = 5.14\,s^{-1}$): ω = [6.5, 7.0, 7.5, 8.0, 8.5, and 9.0] × 5.14 s-1

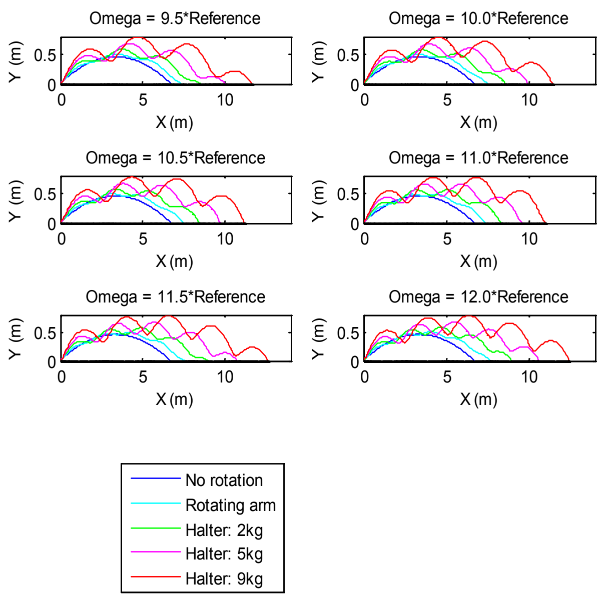

Figure 5. Simulated trajectories of the center of mass based on several angular velocities expressed in terms of the reference value ($\omega_{ref} = 5.14\,s^{-1}$): ω = [9.5, 10.0, 10.5, 11.0, 11.5, and 12.0] × 5.14 s-1

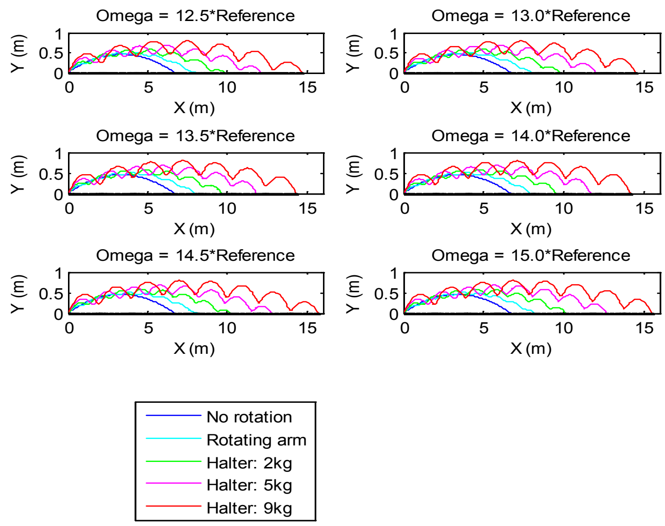

Figure 6. Simulated trajectories of the center of mass based on several angular velocities expressed in terms of the reference value ($\omega_{ref} = 5.14\,\mathrm{s}^{-1}$): ω = [12.5, 13.0, 13.5, 14.0, 14.5, and 15.0] × 5.14 s-1.

One can notice in Fig.2 up to Fig.6 that, although the angular velocity, ω, highly influences the length of jump, its effect is not exactly proportional to L. Therefore, there is need of further investigation on the details.

3.2. Details on Realistic Jumps

Having established a possible theoretical framework for the explanation of the renowned ancient Greek jumps, we can now study in more details the realistic case of approximately half a rotation during the jump. Practically, a jumper can conveniently put his arms in the feasible interval $-100\,(\text{posterior}) \le \theta \le 90\,(\text{anterior})$ degrees, which corresponds to a total of $\Delta\theta$ = 190 degrees, where the angle $\Delta\theta$ is shown in Figure 1. Below, results are shown for typical cases.

Again, for the jump of a non-rotating athlete under the given velocity components (u_0 = 11 m/s, v_0 = 3 m/s), (8) gives the nominal value of jump period T=0.6116 s, while (9) gives the corresponding nominal jump length L = 6.7278 m. When considering rotation of the arms starting from θ_0 = 90 degrees, the aforementioned quantities (L and T) increase according to Table 2. It should become clear that, $\Delta\theta$=90 means that the angular velocity ω has been chosen so as within the anticipated duration T the drawn angle fulfills the condition: $\omega T.(180/\pi) = \Delta\theta = 90$deg. Clearly, when the rotational angle $\Delta\theta$ increases, the corresponding angular velocity ω increases as well. In reality, due to the slight increase in jump duration ($T^* > T$) the actual angle $\Delta\theta^*$ is somehow larger ($\Delta\theta^* > \Delta\theta$) as shown in the bottom part of Table 2 (namely 2c).

The results are summarized as follows:

1) The rotation of the arms (without weights) at an angular velocity ω such as $\omega T=\pi/2$ (approximately 90 degrees) leads to: T^* = 0.6207 s and L^* = 6.8433 m, while the true rotational angle becomes equal to $\Delta\theta^*$ = 91.33 degrees (slightly larger than 90 degrees).

2) The rotation of the arms (weight of 2 kg in each hand) at an angular velocity ω such as $\omega T = \pi/2$ (approximately 90 degrees) leads to: T^* = 0.6334s and L^* = 7.0077 m, while the true rotational angle becomes equal to $\Delta\theta^*$ = 93.21 degrees.

3) The rotation of the arms (weight of 9 kg in each hand) at an angular velocity ω such as $\omega T = \pi/2$ (approximately 90 degrees) leads to: T^* = 0.6709s and L^* = 7.5045 m, while the true rotational angle becomes equal to $\Delta\theta^*$ = 98.72 degrees.

4) The *maximum* length of jump (8.4473m) appears for $\Delta\theta$ = 190 (actually it is $\Delta\theta^*$ = 217.49 degrees, see Table 2c), and especially for the *heavier* halter (9 kg).

5) It can be noticed that when ($\Delta\theta$ = 200 deg) the action of the weight stops to be advantageous.

Furthermore, when performing more than one revolution in the jump duration holding a mass m_h = 9 kg in each hand, for several initial positions, θ_0, the situation changes as shown in Figure 7. In more details, it can be noticed that if the angular velocity ω is chosen so as $200 < \omega T.(180/\pi) = \Delta\theta < 450$deg (it is reminded that T = 0.6116 s), the length of jump becomes smaller than the abovementioned 8.4473 m but it obtains another higher value (9.5118 m) when the angular velocity becomes close to $\omega T(180/\pi)$ = 580 deg (that is 310 degrees after the previous maximum of 190 deg), while a third local maximum (10.5716 m) appears at $\Delta\theta$ = 930 deg, and so on. It is remarkable that the initial position θ_0 = 90 degrees (blue line) always leads to the *maximum* value (at the end of the first half rotation) and has the *best performance* for any amount of weight handheld.

As can be noticed in Figure 8 (again m_h = 9 kg), similar graphs are obtained for θ_0 = 40 and 30 deg, whereas when θ_0 = 20, 10 and 0 deg the graphs progressively and rapidly intersect the horizontal $\Delta\theta$-axis.

In conclusion, when the halteres are rotating forwards (in the clockwise direction) they always contribute to produce jump lengths higher than the reference value of L = 6.7278 m (horizontal black line in Figure 7 and Figure 8).

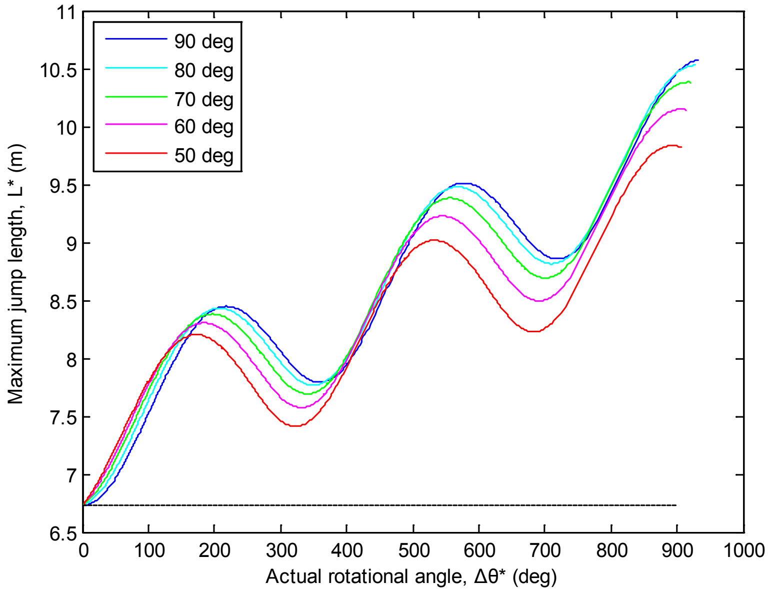

Figure 7. Maximum long jump length versus the actual rotational angle $\Delta\theta^*$ from the most posterior position up to the most anterior (blue, cyan, green, magenta and red colours correspond to initial positions θ_0 = 90, 80, 70, 60 and 50 degrees, respectively); rotation is forwards (in the clockwise direction).

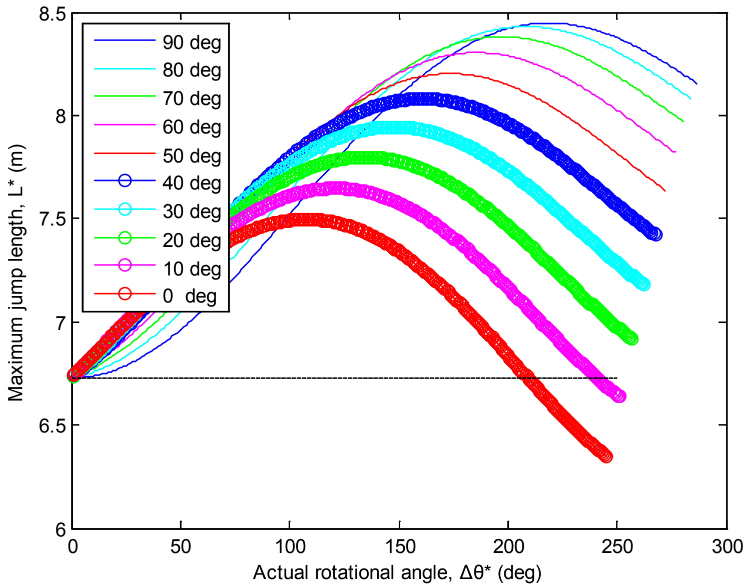

Figure 8. Zoom of Figure 7, in which more initial positions are added (the circled colors: blue, cyan, green, magenta and red correspond to θ_0 = 40, 30, 20, 10 and 0 degrees)

Table 2. Variation of the jump length, jump duration and the actual rotational angle $\Delta\theta^*$ when starting from the vertical posterior position (θ_0=90 deg) and rotating forwards at a constant angular velocity ω of varying magnitude such as $\omega T = \Delta\theta(\pi/180)$ with T=0.6116 s.

(a)

Mass of each halter (kg)	LENGTH OF JUMP: L^* (m)								
	θ_0 = 90 deg								
	$\Delta\theta$ = 70	$\Delta\theta$ = 90	$\Delta\theta$ = 110	$\Delta\theta$ = 120	$\Delta\theta$ = 130	$\Delta\theta$ = 150	$\Delta\theta$ = 180	$\Delta\theta$ = 190	$\Delta\theta$ = 200
m = 0	6.8010	6.8433	6.8889	6.9114	6.9330	6.9709	7.0087	7.0152	7.0187
m = 2	6.9044	7.0077	7.1187	7.1730	7.2243	7.3118	7.3920	7.4037	7.4083
m = 5	7.0457	7.2345	7.4363	7.5332	7.6229	7.7698	7.8894	7.9027	7.9043
m = 9	7.2122	7.5045	7.8135	7.9585	8.0892	8.2925	8.4373	8.4473	8.4418
Non rotating arms	**6.7278 m**								

(b)

Mass of each halter (kg)	DURATION OF JUMP: T^* (s)								
	θ_0 = 90 deg								
	$\Delta\theta$ = 70	$\Delta\theta$ = 90	$\Delta\theta$ = 110	$\Delta\theta$ = 120	$\Delta\theta$ = 130	$\Delta\theta$ = 150	$\Delta\theta$ = 180	$\Delta\theta$ = 190	$\Delta\theta$ = 200
m = 0	0.6176	0.6207	0.6237	0.6251	0.6264	0.6282	0.6290	0.6288	0.6282
m = 2	0.6259	0.6334	0.6407	0.6439	0.6467	0.6505	0.6513	0.6503	0.6488
m = 5	0.6373	0.6507	0.6635	0.6690	0.6734	0.6788	0.6779	0.6755	0.6724
m = 9	0.6505	0.6709	0.6897	0.6973	0.7030	0.7088	0.7043	0.7001	0.6950
Non rotating arms	**0.6116 s**								

(c)

Mass of each halter (kg)	ACTUAL ROTATIONAL ANGLE: $\Delta\theta^*$ (deg)								
	θ_0 = 90 deg								
	$\Delta\theta$ = 70	$\Delta\theta$ = 90	$\Delta\theta$ = 110	$\Delta\theta$ = 120	$\Delta\theta$ = 130	$\Delta\theta$ = 150	$\Delta\theta$ = 180	$\Delta\theta$ = 190	$\Delta\theta$ = 200
m = 0	70.68	91.33	112.18	122.65	133.13	154.07	185.12	195.33	205.43
m = 2	71.64	93.21	115.23	126.34	137.46	159.54	191.69	202.03	212.16
m = 5	72.93	95.75	119.34	131.25	143.14	166.47	199.50	209.85	219.87
m = 9	74.45	98.72	124.05	136.80	149.43	173.83	207.26	217.49	227.27

Finally, if backwards (anti-clockwise) direction is considered, the results of Figure 9 are derived for the interval $0 < \Delta\theta < 270$ deg. One can notice that the backwards rotation leads to either a small improvement or mostly to a decrease of the jump length. Again, the horizontal dotted line corresponds to the reference value of L = 6.7278 m.

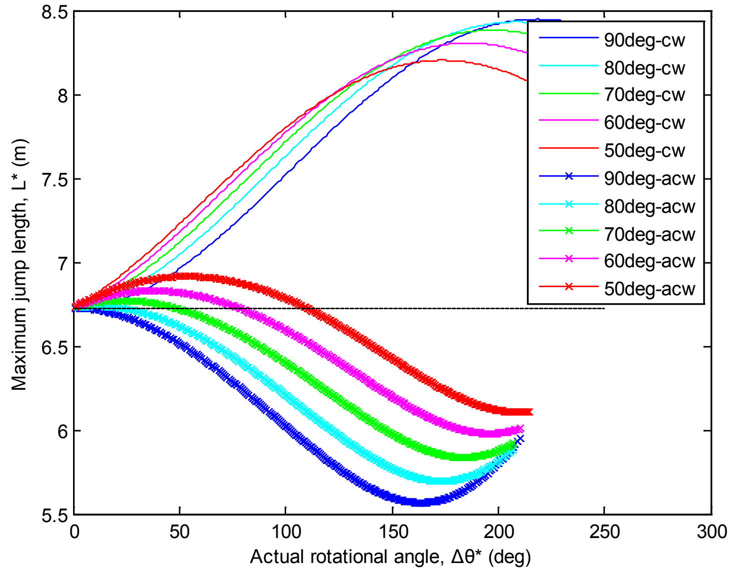

Figure 9. Comparison between forwards (cw: clockwise, upper part) and backwards (acw: anti-clockwise, lower part) concerning the maximum length of jump versus the actual rotational angle $\Delta\theta^*$ (blue, cyan, green, magenta and red colors correspond to initial positions θ_0 = 90, 80, 70, 60 and 50 degrees, respectively). Regular lines correspond to forwards rotation (posterior-to-anterior) whereas thick crossed lines (×) correspond to backwards rotation (anterior-to-posterior)

3.3. Sensitivity Analysis

Concerning the (initial) angle at take-off, θ_0, (see, Fig. 1), its effect on the length of jump was previously illustrated in both Fig. 7 and Fig. 8. However, these results were obtained for the particular case in which the weight of the halter in each hand was equal to m_h = 9 kg.

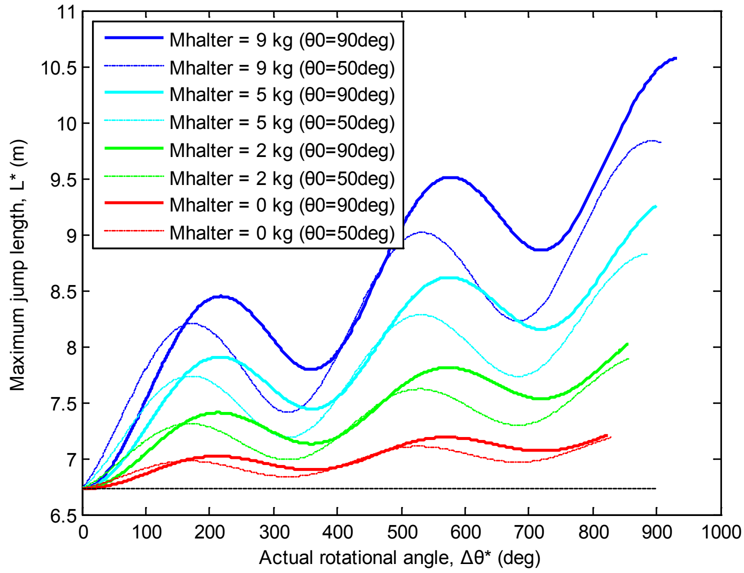

Figure 10. Maximum long jump length versus the actual rotational angle $\Delta\theta^*$ for several halter weights (0, 2, 5 and 9 kg), as well as two discrete arm positions (θ_0 = 90, 50 degrees) at take-off; rotation is forwards

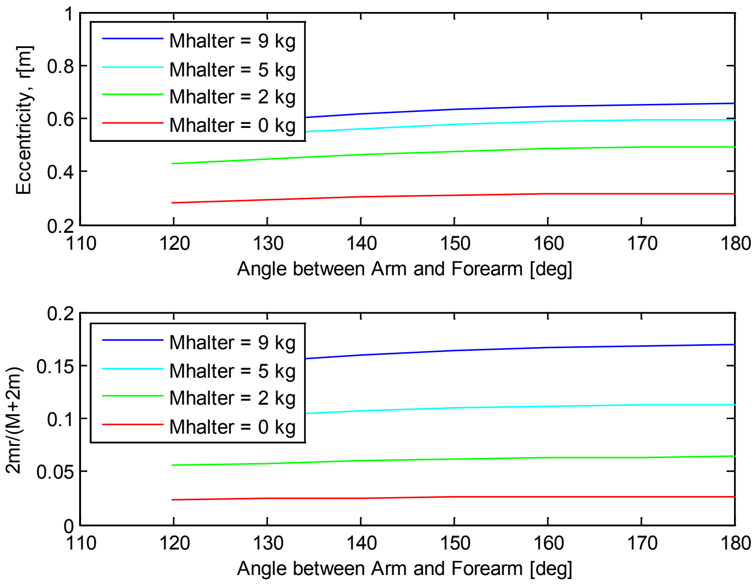

Figure 11. The influence of the angle between arm and forearm (at elbow) on (i) the eccentricity, r, of the center of arm for the entire arm [Top], as well as (ii) on the coefficient $2mr(M+2m)$ [Bottom]

In the sequence, we perform a sensitivity analysis to see whether the same tendency appears for smaller halter weights. In fact, Fig. 10 shows that *the more heavy the halter the greater the length of jump*. Moreover, with some minor exceptions that appear for small values of the actual rotational angle, $\Delta\theta^*$, *the larger the take-off angle* (θ_0=90deg: thickest lines) *the greater the length of jump*.

Remark: It is worthy to explain that the above-mentioned small values of the angle between take-off and landing, $\Delta\theta^*$, (when unfortunately occur) correspond to small values of angular velocity, ω, and also to large values of linear momentum of the halters at landing in the forward horizontal direction; in other words, a small rotation of the arms does not allow for the linear momentum to pass from the halter to the jumper's body. In contrast, when for example θ_0=90deg, the linear momentum of halters at take-off is horizontal and forwards, and if the angular velocity is chosen so as the angle $\Delta\theta^*$ be about 180 degrees, the horizontal component of linear momentum of the halter at landing is backwards; therefore, due to the momentum conservation the jumper's body obtains this difference of momenta, that is between forward (at take-off) and backward (at landing) ones.

3.4. The Effect of Anthropometry of Human Body

So far we have considered a typical jumper's body for which the angle between the arm and forearm equals to α = 180 degrees; in other words, the entire arm was assumed to be ideally straight. If, however, the before-mentioned angle is smaller than 180 degrees (α < 180), then (10) is no further valid as is. Based on the anthropometry of the human body (the weights of all three: arm, forearm and hand were split according to [14, p.3], and the location of centers of mass was expressed as a ratio of the distance from the proximal end according to [14, p.5]), the eccentricity r is easily found to depend on both the angle α, as well as on the halter weight, m_h, as shown in Fig. 11 (top). In the bottom of the same figure one can also found the result of calculations of the critical quantity, $2mr/(M+2m)$, which appears in (11); for the latter, the halter's weight, m_h, is seen to be more critical than what the elbow angle, α, is.

4. Discussion

There are four main components of the long jump: the approach run, the last two strides, takeoff, action in the air, and landing. Speed in the run-up, or approach and a high leap off the board are the fundamentals of success. Because speed is such an important factor of the approach, it is not surprising that many long jumpers also compete successfully in sprints.

Nevertheless, in addition to the initial velocity, another factor is the impulse produced by the moving members, which is related to additional energy released from jumper's chemical energy.

Previous studies concern mostly the standing jump and suggest that "halteres were swung back and forth by the jumper before take-off, thrust forwards during the first part of the flight, and finally swung backwards just before landing, as depicted in a variety of vase paintings" [9]. Most of the evidence concerning the jumping technique in ancient Greece is in favor of the standing long jump (multiple jumps).

Although the previous writings are absolutely reasonable, however the assumption of a *constant angular velocity* used in this study (like a mechanical motor) offers a convenient tool of thoughts for commenting on the significance of the synchronization of the rotating arms on the achieved jump length (similar thoughts can be also extended to the rotating legs).

In brief, the results of this study suggest that:

1) When halteres are used for the highest possible (in the sense of jumper's comfort) angular path ($\Delta\theta$ = 190 degrees owing the highest angular velocity) the maximum jump length is achieved.

2) Forwards rotation is advantageous while backwards one is disadvantageous.

3) Whatever the initial position θ_0 of the halteres at take-off is, the forwards rotation leads to an increased jump length. For a specific angular velocity, the higher the weight of each halter, the longest the jumper remains in the air and the longest the jump is.

4) The true reason of the increased length of jump is that the arm rotation causes an increase in the take-off velocity of the center of mass of the *entire* jumper's body (including arms): see Appendix A.

As mentioned above, it is not sufficient only to increase the rotation of the arms but to put them at a proper position at the instance of take-off. In general, the 'secret' point is to start with linear momentum of the halter, which is fully given to increase the linear momentum of jumper's body.

It is worthy to mention that in mid-1950s a very similar idea was patented by Norman Dean (a civil service employee residing in Washington DC) who proposed the use of two contra-rotating eccentric masses in order to convert rotary motion to unidirectional motion; see also: http://en.wikipedia.org/wiki/Dean_drive. A detailed mechanical analysis of the latter renowned mechanism in the vertical direction has been recently reported [11-13]; there it was found that the maximum length of *vertical* trajectory is achieved only when the initial position of the rotating rods is the *horizontal* one. The aforementioned finding was extended in this paper as follows: In order to achieve the maximum length of jump the rotating arms must be perpendicular to the jumper's motion.

The weaknesses of the proposed model are as follows:

1) The action of the air was not taken into consideration.

2) The angular velocity, from take-off until landing, was considered to be constant. Patterns of variable angular velocity, although possible to be treated, are not included in this study.

3) The body of the jumper (of mass *M*) was assumed to be rigid and non rotating; therefore the action of the legs and the rotation of the torso were not considered.

4) Body changes were not considered. It is reminded that except of the horizontal length of points along the parabolic trajectory (as used in this paper), we had to add (i) the horizontal distance between the front edge of the take-off board and the position of the jumper's center of gravity at the moment of take-off, as well as (ii) the horizontal distance between the jumper's center of gravity at the instant the jumper makes contact with the ground on landing and the mark on the landing area that is used to define the overall length of the jump (details can be found in [2]).

5) The length of jump refers to the motion of the center of mass of the abovementioned rigid mass *M*. In practice, the center of mass of the body is about 0.9m above the ground level and must be taken into consideration.

6) The magnitude of the induced forces in the joints ($m\omega^2 r$) has not been calculated. However, even if they are high enough (particularly at high angular velocities), future experiments could be conducted using suitable *exoskeletons* to avoid harmful effects.

7) No experimental measurements have been performed. It is suspected that taking off at 11.0 m/s with heavy weights of 9 kg (however mentioned in literature) in each hand is probably outside the human capacities thus not possible, nor is a safe landing after a flight phase of 15 meters. Also, multiple rotations in the airborn phase are rather impossible to accomplish, and if so, they would make it impossible to perform a landing in balance.

Despite the above weaknesses, this study is a thorough investigation on the consequences of arm rotation and can be easily extended from jumps with run-up, as those studied in this paper, to ones from standstill (namely, to standing long jumps) where experimental data are available [10]. Also, the main idea of this paper can be easily incorporated into more advanced third party biomechanical models in which full dynamics of the long jump is considered [15].

5. Conclusion

This study extends the idea of the double-arm style of take-off, which works by moving both arms in a vertical direction as the competitor takes off. So far it was known that the latter produces a high hip height and a large vertical impulse. Based on theoretical considerations, this study shows that when halteres rotate forwards they always achieve an increase of the length of jump. Therefore, if ancient Greeks could technically apply extremely high angular velocities, such as 12 rotations of their straightened arms during the period of the jump, from a theoretical point of view this hypothetical event would lead to a length of jump of the order of 15 meters or still higher. Alternatively, the weights could be driven by the jumper to follow circumferences of smaller radius but then a still larger number of rotations than twelve would be necessary in order to produce an equivalent action.

APPENDIX

Justification of the mechanical behavior

In the simulations, the velocity (speed and direction) of the jumper's body is always the same, and the angular velocity of the jumper's arms at the instant of takeoff and the mass of the halteres are increased. Increasing the angular velocity of the arms or the mass of the halteres means that the velocity of the jumper's centre of mass at the instant of takeoff (i.e. body + arms + halteres) is different in each simulation. That is, in the simulations where the arms and halteres are rotating very fast clockwise at takeoff, the takeoff velocity of the athlete's center of mass is actually much greater than when the jumper's arms+halteres are not rotating. The increase in the takeoff velocity of the jumper's centre of mass is the true source of the increase in jump distance in these simulations.

The velocity of the jumper's center of mass at the instant of takeoff can be calculated from the vector sum of the linear momentum of the jumper's body at the instant of takeoff and the linear momentum of jumper's arms and halteres at the instant of takeoff.

When the jumper has no halteres (i.e. m = 3.2 kg) and the arms are not rotating (ω = 0), the linear momentum of the athlete's center of mass at the instant of takeoff is:

Horizontal momentum: (70 kg × 11 m/s) + (6.4 kg × 11 m/s)

Vertical momentum: (70 kg × 3 m/s) + (6.4 kg × 3 m/s)

Total momentum: 76.4 kg at 11.4 m/s and 15.3 degrees to horizontal.

That is, the takeoff velocity of the athlete's center of mass is 11.4 m/s at 15.3 degrees, and therefore the jump distance is 6.75 m.

Now consider the athlete when the arms are rotating clockwise at 1.0 times the reference value (ω = 5.14 rad/s) with 9 kg halteres (i.e. m = 12.2 kg). The linear momentum of the athlete's center of mass at the instant of takeoff is then

Horizontal momentum: (70 kg × 11 m/s) + [2 × 12.22 kg × (11 m/s + 5.14 rad/s × 0.65 m)]

Vertical momentum: (70 kg × 3 m/s)

Total momentum: 76.4 kg at 14.92 m/s and 10.62 degrees to horizontal.

That is, the takeoff velocity of the jumper's center of mass is 14.92 m/s at 10.62 degrees, and therefore the jump distance is 8.23 m.

REFERENCES

[1] M. Lenoir, D. De Clercq, W. Laporte. The "how" and "why" of the ancient Greek long jump with weights: A five-fold symmetric jump in a row? Journal of Sports Sciences, Vol.23, 1033-1043, 2005.

[2] A. J. Ward-Smith. The application of modern methods of biomechanics to the evaluation of jumping performance in ancient Greece, Journal of Sports Sciences, Vol.13, 223 – 228, 1995.

[3] E. N. Gardiner. Phayllos and his record jump, Journal of Hellenic Studies, Vol.24, 70 – 80, 1904.

[4] E. N. Gardiner. Further notes on the Greek jump, Journal of Hellenic Studies, Vol.24, 179 – 194, 1904.

[5] W. W. Hyde. The pentathlon jump, American Journal of Philology, Vol.236, 405 – 418, 1938.

[6] R. L. Howland. Phayllos and the long jump record, Proceedings of the Cambridge Philological Society, Vol.181, 31, 1950.

[7] J. Ebert. Der Pentathlonsprung, Abh. Sochs. Akad. Wiss. Philo-Hisot. Klasse, Vol.56, 35 – 65, 1963.

[8] R. K. Barney. The ancient Greek pentathlon jump: A preliminary reinterpretive examination. In: Proceedings of the International Congress on Physical Activity Sciences (pp. 279 – 288). Quebec: International Congress on Physical Activity Sciences, 1976.

[9] A. E. Minetti, L. P. Ardigó. Halteres used in ancient Olympic long jump, Nature, Vol.420, 141 – 142, 2002.

[10] C. Papadopoulos, G. Noussios, E. Manopoulos, O. Kiritsi, G. Ntones, E. Gantiraga, I. Gissis. Standing long jump and handheld halters; is jumping performance improved? Journal of Human Sport & Exercise, Vol.6, No.2, 436-443, 2011.

[11] C. G. Provatidis. Some issues on inertia propulsion mechanisms using two contra-rotating masses, Theory of Mechanisms and Machines, Vol.8, No.1, 34-41, 2010.

[12] C. G. Provatidis. A Study of the Mechanics of an Oscillating Mechanism, International Journal of Mechanics, Vol.5, No.4, 263-274, 2011.

[13] C. G. Provatidis. An overview of the mechanics of oscillating mechanisms, American Journal of Mechanical Engineering, Vol.1, No.3, 58-65, 2013.

[14] C. E. Clauser, J. T. McConville, J. W. Young. Weight, Volume and Center of Mass of Segments of the Human Body, AMRL-TR-69-70, Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio, 1969.

[15] A. Seyfarth, A. Friedrichs, V. Wank, R. Blickman. Dynamics of the long jump, Journal of Biomechanics, Vol.32, 1259-1267, 1999.